COMBINATION OF DETERMINISTIC AND STOCHASTIC APPROACHES TO THE IMAGE RECONSTRUCTION

J. Dědková

Ústav teoretické a experimentální elektrotechniky,FEKT VUT v Brně
Kolejní 4, 612 00 Brno, tel.: +420 541 149 523, e-mail: dedkova@feec.vutbr.cz

Summary In this paper is described a new algorithm based on the combination of deterministic and stochastic approaches to the reconstruction process of the surface conductivity distribution to obtain the best results. The images of the electrical surface conductivity distribution can be reconstructed from voltage measurement captured on the boundaries of an object. The image reconstruction problem is an ill-posed inverse problem of finding such surface conductivity that minimizes the suitable optimisation criterion. The advantages of a new approach are compared with properties of deterministic and stochastic approaches during the same image reconstructions. It will be shown that proposed algorithm is a very effective way to obtain the satisfying identification of cracks in special structures called honeycombs.

1. INTRODUCTION

Electrical impedance tomography (EIT) is used to reconstruct the conductivity distribution by the measured surface electric potential distribution around the phantom when injecting current into the object [1]. The electric potential distribution on the surface generated by the injected current could be obtained as a solution of the Laplace equation. Many published papers have already described the recovering of volume conductivity. But it is well known that some industrial products can better be described by surface conductivity or by the combination of both parameters.

2. BASIC THEORY

The theoretical background of EIT with surface conductivities is given in [2]. EIT image reconstruction problem is an ill-posed inverse problem of finding such surface conductivity σ_s that minimizes certain optimisation criterion, which can be given by the suitable objective function. Let define the primal objective function

$$\Psi(\sigma_s) = \frac{1}{2}\sum \left\| \boldsymbol{U}_{\mathrm{M}} - \boldsymbol{U}_{\mathrm{FEM}}(\sigma_s) \right\|^2 \tag{1}$$

Here σ_s is the surface conductivity distribution vector in the object, U_M is the vector of measured voltages on the boundary, and $U_{FEM}(\sigma_s)$ is the vector of computed peripheral voltages in respect to σ_s, which can be obtained using FEM. To minimize the objective function $\Psi(\sigma_s)$ we can use a lot of different methods based on both deterministic and stochastic approaches [3, 4, 5, 6]. When we apply the deterministic method based on the Least Squares (LS) method due to the ill-posed nature of the problem, regularization has to be used. It is possible to apply the widely known Tikhonov Regularization Method (TRM), described in [4]. With respect to regularization the primal object function of TRM can be written in the form

$$\Psi(\sigma_s) = \frac{1}{2}\sum \left\| \boldsymbol{U}_{\mathrm{M}} - \boldsymbol{U}_{\mathrm{FEM}}(\sigma_s) \right\|^2 + \alpha \left\| L\sigma_s \right\|^2 \tag{2}$$

Here, α is the regularization parameter, and L is the so-called regularization matrix. For the solutions of (2) we can apply a Newton-Raphson method. The iterative procedures are likely to be trapped in local minima and so sophisticated regularization must be taken into account to obtain the stable solution.

A little bit different approach present global optimizing evolutionary algorithms, such as genetic algorithms, which have been recently applied to the EIT problem [3, 5]. We have been applied to the reconstruction of the conductivity distribution the algorithm based on the Controlled Selection of Non-homogeneities (CSN) described in [7]. The optimization of the primal objective function (1) based on the CSN is a relatively new technique with a very simple basic principle, but it is unfortunately a very time-consuming algorithm.

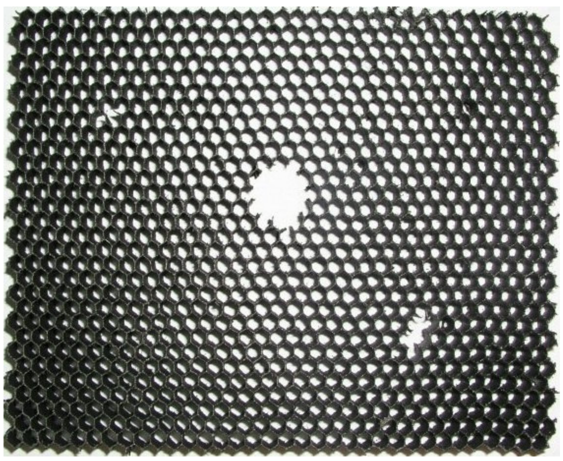

Fig. 1. Honeycomb structure

To obtain the effective method for the image reconstruction process we propose algorithm based on the both above mentioned methods. First we apply the TRM to find subregions where the non-homogeneities can occur and then it is possible to specify effectively the accurate distribution of the non-homogeneities using the CSN.

The proposed algorithm can be used to the practical detection of cracks in honeycomb structures, see Fig. 1.

3. VERIFICATION RESULTS

Two simple examples of 2D grid of honeycombs structures are given in Fig. 2 (respectively in Fig. 4). The mesh has a total of 384 edges and 272 nodes, (respectively 2400 edges and 1640 nodes).

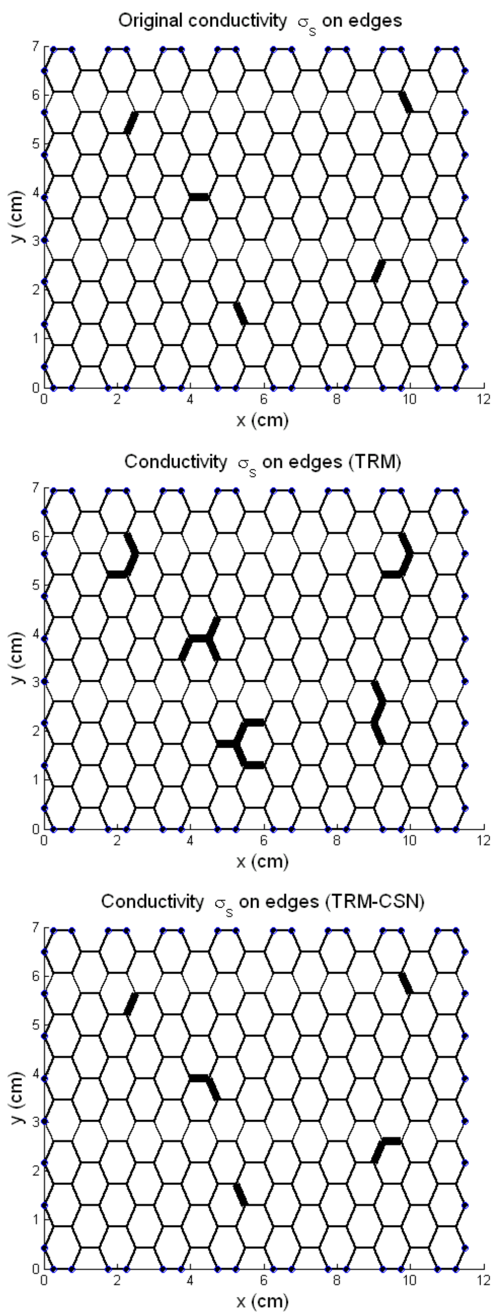

Fig. 2. Example 1 – edge length 1cm

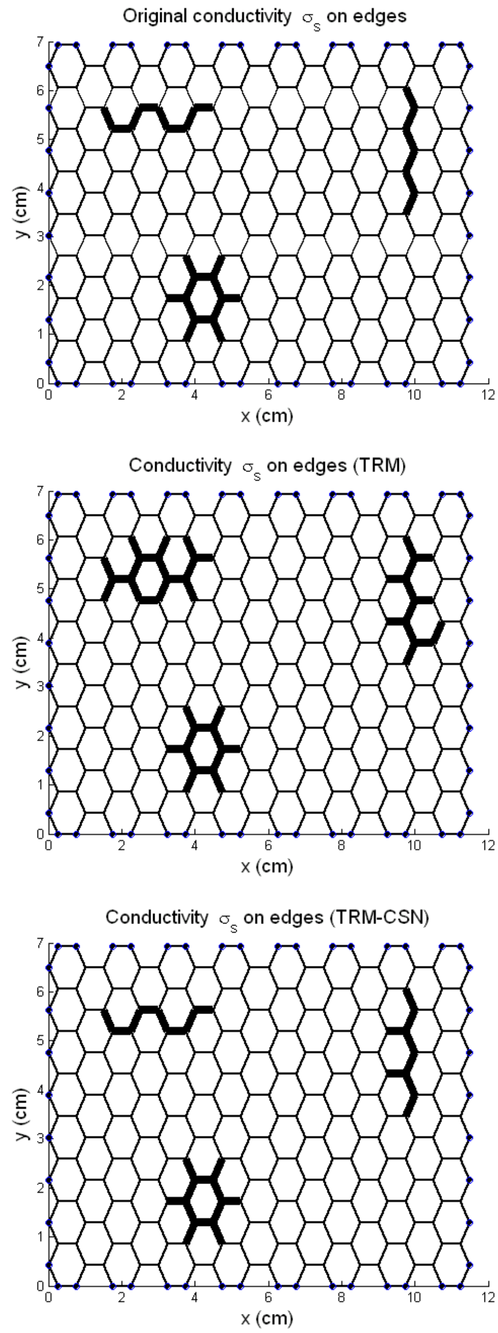

Fig. 3. Example 2 – edge length 1cm

The surface conductivity σ_s has non-zero value 72 700 S (on the black edges), on the black bold marked edges are assume zero value of conductivity and these edges represent some cracks in honeycombs structure (see the original conductivity distributions in Fig. 2 - Fig. 5). In the Fig. 2, Fig. 3 are presented conductivity σ_s distributions on edges obtained using the TRM, CSN and TRM-CSN algorithms for two cases of cracks distribution. When we used the TRM we needed 5 iterations to obtain presented results and the accuracy and the stability was strong depending on the value of the regularization parameter α.

The best results of the reconstruction we obtain using the CSN algorithm (the same as original), but the needed time was ten times greater then the time needed for TRM algorithm for an arrangement due to Fig. 2, Fig. 3 and more then three hundred times greater for an arrangement due to Fig. 4, Fig. 5. When we used to reconstruction proposed algorithm TRM-CSN, we obtained the satisfactory reconstruction results in the same time which is needed for the TRM.

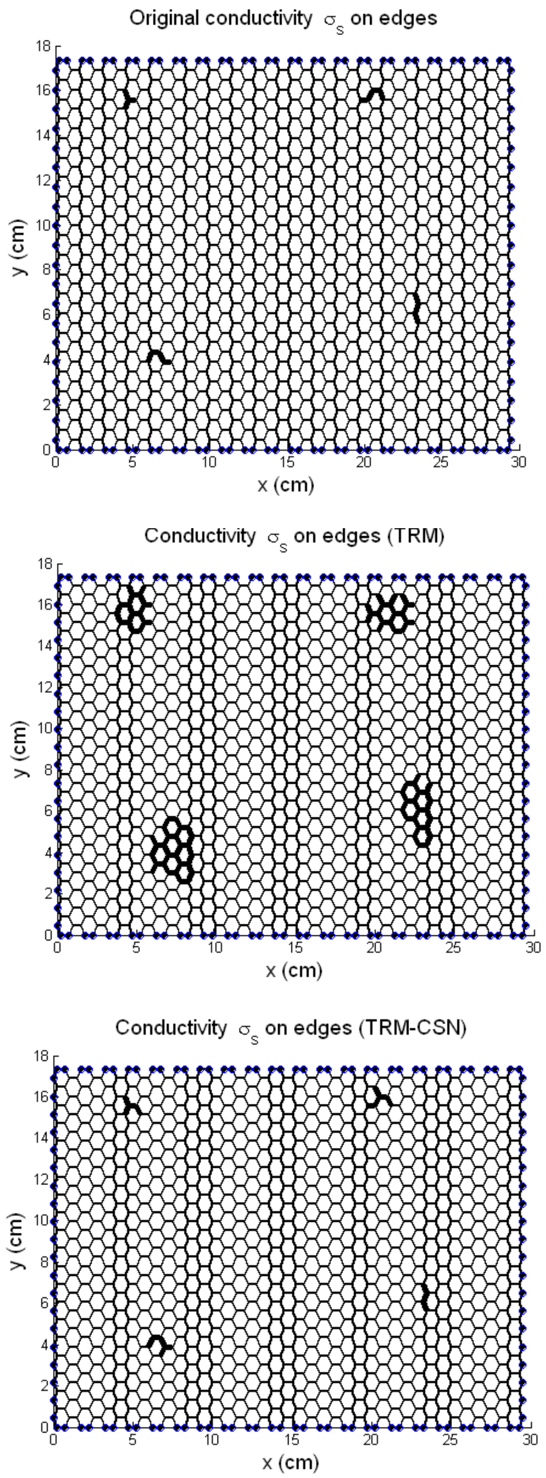

Fig. 4. Example 2 – edge length 0.5 cm

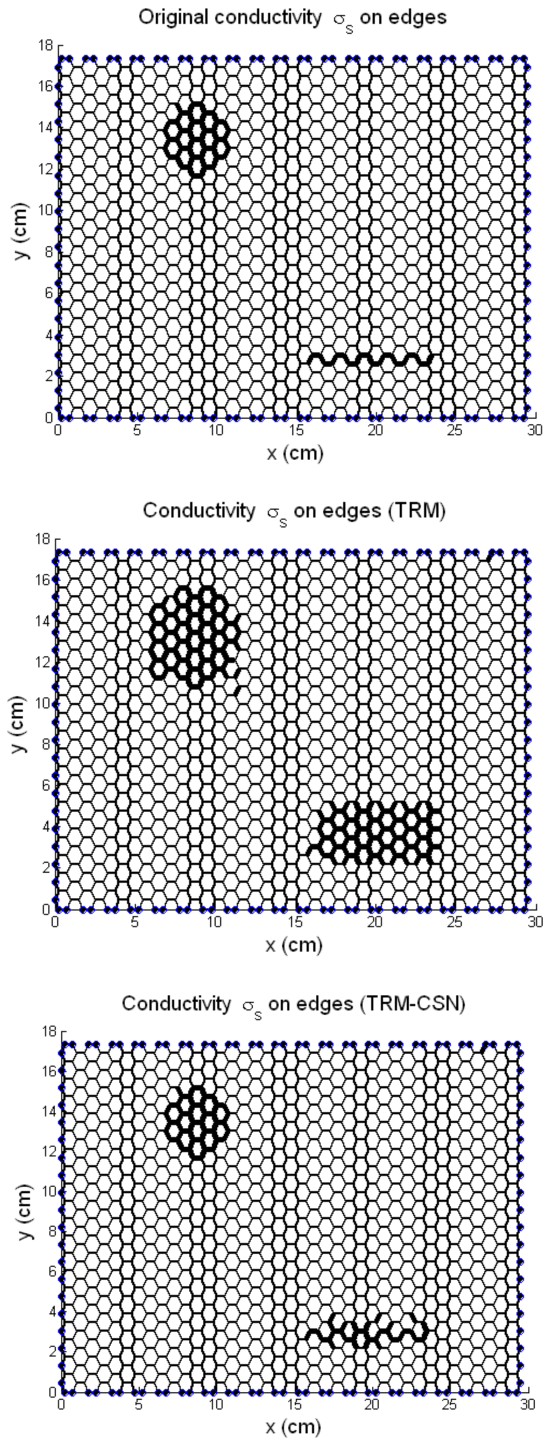

Fig. 5. Example 2 – edge length 0.5 cm

4. CONCLUSION

In this paper has been presented one effective algorithm to a reconstruction of the cracks distributions in special materials structures. Many numerical experiments performed during the above-described methods have resulted in the conclusion that the application of the CSN reconstruction algorithm has a significant advantage over the TRM in better accuracy and the stability of the reconstruction process, but on the other hand the CSN is a very time-consuming technique. The algorithm TRM-CSN based on combination of the both methods was proposed and verified. On the basis of many numerical experiments, we can confidently say that the TRM-CSN algorithm is a very effective tool to detect the cracks distributions in honeycombs structures with respect to the best accuracy, the stability and the space resolution. The results stated above as well as many other examples were obtained using a program written in MATLAB by author.

The next work will focused to enlarge the possibility to reconstruction of the cracks distributions in 3D honeycomb structures.

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