INVESTIGATION OF MAGNETIC FIELD IN THE SUBWAY STATION

C. Henzl, P. Kačor, J. Paleček

VŠB-TU Ostrava, 17. listopadu 15, Ostrava-Poruba, 708 33
e-mail:ctibor.henzl@vsb.cz

Summary Paper deals with computing of magnetic field in the rails surroundings of subway (underground). The calculation of field is made by using of an analytical method in Excel VBA. There is also used commercial FEM software ANSYS and finite element method. Output results of the both methods are finally compared.

1. INTRODUCTION

In the calculation of the electromagnetic field one may meet with the so-called incommensurable problems when the relevant part of the defined area is much less than the remaining defined area.

This occurs in the case when is necessary to determine the magnetic field created by systems of current carrying wires.

The field should be determined very close to the wires, but at the same time it is necessary to appreciate where the field can not be omitted; where its value is given and inconsiderable.

This task can be solved analytically in some special cases, when the surrounding medium is homogenous, non-ferromagnetic and non-conducting. The appropriate integral formulas are used. On the other side, in more common cases, when the surrounding medium is non-homogenous, it is necessary to formulate the corresponding differential equations and solve them by means of a suitable numerical algorithm.

The present contribution compares the mentioned numerical method with a simple integral model; at the same time the typical incommensurable problem is taken into account - distribution of the magnetic field in the surroundings of the electric current system for electric traction.

In the following sections we define the problem to solve, the mathematical models to be used, and the results obtained which are discussed and compared.

2. FORMULATION OF THE PROBLEM

The problem to solve is the calculation of the magnetic field near the line of subway (underground) in Prague. Schematic representation is depicted in Fig. 1. Types of conductors for the wires and assumed current distribution are presented in Tables 1, 2 and 3.

It is assumed that no vehicles are present for the calculations.

Current flowing in each conductor is determined from Table 1, 2 and 3. The current of the supply rail is 2000 A and the rail's reverse current is 800 A.

The geometry of the problem is two dimensional, in the *x,y* co-ordinate system (differential model), or one dimensional in cylindrical *r,z* co-ordinate system (integral model).

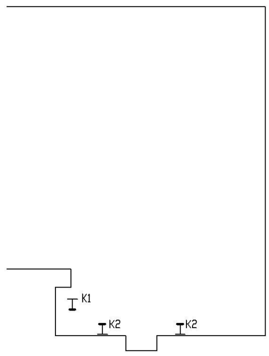

Fig. 1 Scheme of line in station
K1-*Supply rail*; K2-*Curry rail*

Tab. 1. Types of conductors

Conductor	Current carrying cross section (mm^2)	Height upper crown of the rail (mm)	Distribution of traction current between rails (%)	Distance from the centre of area (mm)
Supply rail	6297	170	100	1300
Curry rail	6297	0,0	40	717,5
Curry rail	6297	0,0	40	717,5

Tab. 2. Distribution of the current between rail and earth

Current of traction rail (A)	2000	
Assumed distribution of the current between rail and earth(%)	Rail	80
	Earth	20

Tab. 3. Parameters of the rail

Type of the rail	Cross – section (mm^2)	Wheel track
S49	6297	1435

3. MATHEMATICAL MODELS

3.1 Differential model

The differential equation of problem under investigation has the general form

$$\mathrm{rot}\frac{1}{\mu}\,\mathrm{rot}\boldsymbol{A} = \boldsymbol{J}_{sc} \tag{1}$$

where: $\boldsymbol{J}_{sc}$ is the current density of I_{sc} currents, flowing along single conductors, as described in Section 1. Their value can be expressed as:

$$\mathrm{mod}\boldsymbol{J}_{sc} = \frac{I_{SC}}{S_I} \tag{2}$$

Due to the fact that the current density vector $\boldsymbol{J}_{sc}$ has only a non null component in the $\mathbf{z}_o$ direction, we have:

$\boldsymbol{J}_{sc} = \mathbf{x}_o.0 + \mathbf{y}_o .0 + \mathbf{z}_o J_{sc,z}$, and consequently

$\boldsymbol{A}_{sc} = \mathbf{x}_o.0 + \mathbf{y}_o.0 + \mathbf{z}_o A_z(x,y)$.

The next differential equations are valid for fields $\boldsymbol{J}_{sc}$ and $\boldsymbol{A}_{sc}$

For cross-sections of single conductors

$$\frac{\partial^2 A_z}{\partial x^2} + \frac{\partial^2 A_z}{\partial y^2} = -\mu_0 J_{sc,z} \tag{3}$$

For the surrounding non-conducting medium

$$\frac{\partial^2 A_z}{\partial x^2} + \frac{\partial^2 A_z}{\partial y^2} = 0 \tag{4}$$

Equations (3) and (4); together with the respective boundary conditions can be solved both by means of the method of finite differences or the finite element method.

3.2 Integral model

3.2.1. Vector magnetic potential of direct current wires

Let us choose a cylindrical co-ordinate system with axis z is coincident with the wire orientation of z axis is that of the current I and co-ordinate lines α are oriented clockwise towards z axis. The vector magnetic potential is calculated from its definition

$$B = B_\alpha(r) = \mathrm{rot}_\alpha \boldsymbol{A} = -\frac{dA_z}{dr}; A_z = A_z(r) \tag{5}$$

According to the well known relation for magnetic flux density created by a long straight wire

$$B = \mu_0 H = \frac{\mu_0 I}{2\pi}\frac{1}{r} \tag{6}$$

Therefore

$$A_z = -\int B dr = -\frac{\mu_0 I}{2\pi}\int \frac{dr}{r} = -\frac{\mu_0 I}{2\pi}\ln r + K \tag{7}$$

where K is a constant, which is determined by means of scaling of the vector potential; i.e. localization of places where $A = 0$. If we take $A_z = 0$ for $r = 1$, then we have

If $r < 1$ then $\ln r < 0$ and $A_z > 0$;

If $r > 1$ then $\ln r > 0$ and $A_z < 0$, See [1]. After re-writting r in Cartesian co-ordinates we have

$$K = 0; \quad A_z = -\frac{\mu_0 I}{2\pi}\ln r \tag{8}$$

$$A_z = -\frac{\mu_0 I}{2\pi}\ln\sqrt{x^2 + y^2} \tag{9}$$

(See Fig. 3).

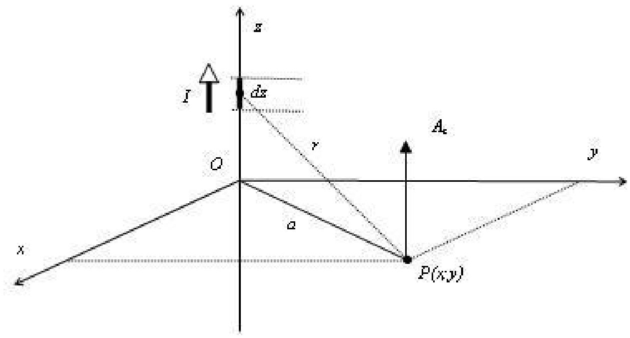

Fig. 3 Current filament in z axis and vector magnetic potential

The expression (9) has to be re-written if the "wire" is more generally situated – parallel to z axis. See Fig. 4 where $[x',y']$ are co-ordinates of conductor with current and $[x,y]$ are co-ordinates of place where vector magnetic potential is calculated.

$$A_z = -\frac{\mu_0 I}{2\pi}\ln\sqrt{(x-x')^2 + (y-y')^2} \tag{10}$$

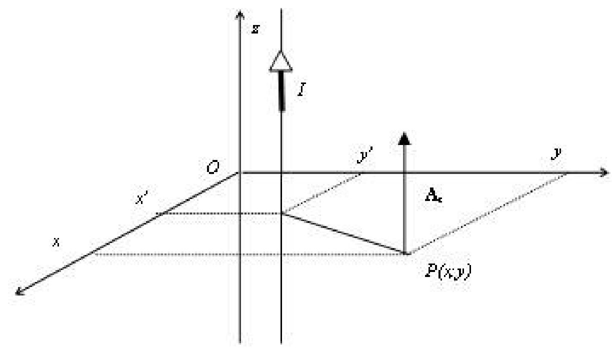

Fig. 4 Current filament parallel to z axis and vector magnetic potential

3.2.2. Magnetic flux density

The magnetic flux density B is calculated according to (11) and expressed in Cartesian coordinates in equations (12) and (13). The final formula (14) is the well known as Ampere's law.

$$\boldsymbol{B} = \mathrm{rot}\boldsymbol{A} \tag{11}$$

$$B_x = \frac{\partial A}{\partial y} = -\frac{\mu_0 I}{2\pi} \cdot \frac{y - y'}{(x - x')^2 + (y - y')^2} \tag{12}$$

$$B_y = -\frac{\partial A}{\partial x} = -\frac{\mu_0 I}{2\pi} \cdot \frac{x - x'}{(x - x')^2 + (y - y')^2} \tag{13}$$

$$B = \sqrt{B_x^2 + B_y^2} = \frac{\mu_0 I}{2\pi a} \tag{14}$$

4. CALCULATIONS

4.1 Results of analytical solution

The results of applying the integral equation model from (5) till (14) are summarized in the following pictures; see from Fig. 5 till Fig. 9.

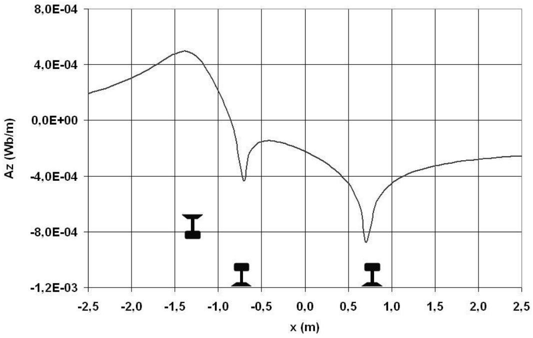

Fig. 5 Vector magnetic potential at the level of rails

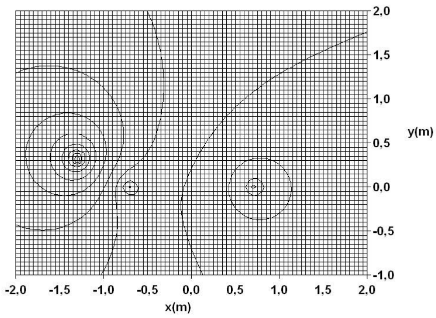

Fig. 6 Equipotential lines of the vector magnetic potential close to the rails – image detail

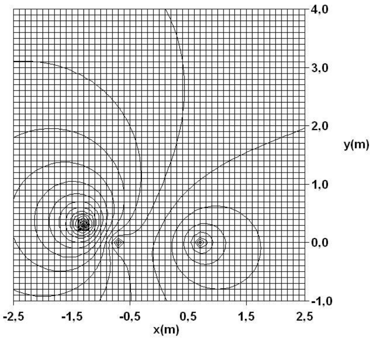

Fig. 7 Equipotential lines of the vector magnetic potential in all the investigated area

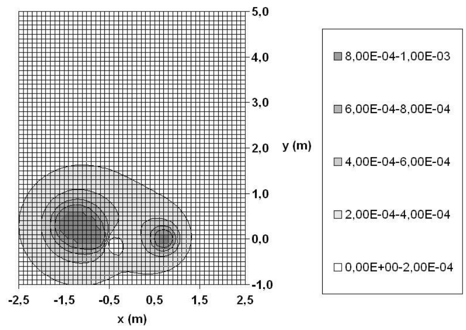

Fig. 8 The map of the magnetic flux density B

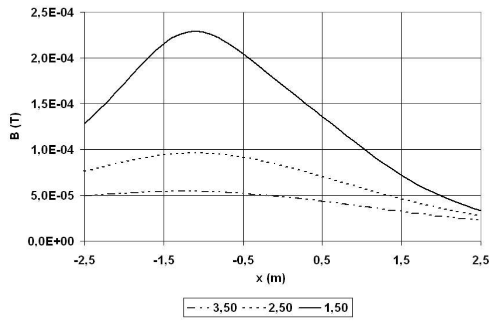

Fig. 9 The magnetic flux density in horizontal levels 3,5 m, 2,5 m and 1,5 m above foot of the rail

5. FEM SIMULATION IN ANSYS

We used ANSYS software and finite element method for solution the same problem and also for comparison of results with previous analysis made by EXCEL VBA.

Basic model is represented by 2-D geometry and identical dimensions.

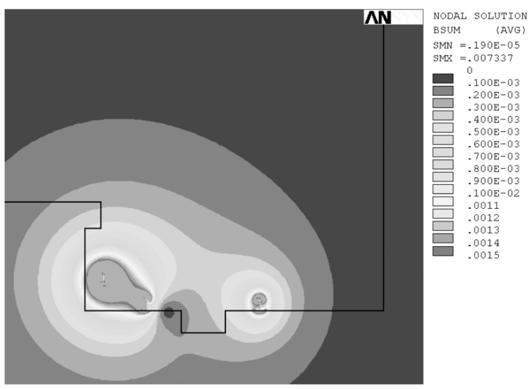

Fig. 10 The map of the magnetic flux density B

The Fig.10 shows distribution of magnetic flux density in model of the underground subway station. There is relative good agreement of values of the magnetic field.

Fig.11 represents flux lines around sources of magnetic field and the chart on Fig.12 shows magnetic flux density in horizontal levels above foot of the rail.

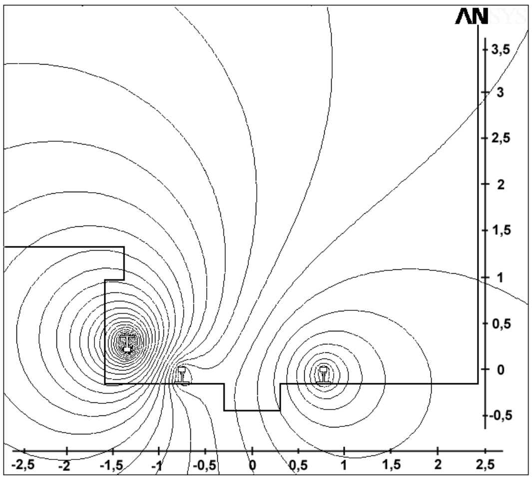

Fig. 11 Flux lines in all the investigated area

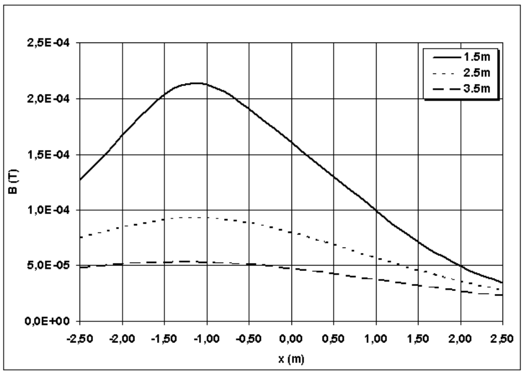

Fig. 12 The magnetic flux density in horizontal levels 3,5 m, 2,5 m and 1,5 m above foot of the rail

6. CONCLUSION

Calculated value of the magnetic field in surroundings of subway station is in the range of mT.

Used analyses show the good agreement of the both methods of solution. The correlations coefficients between single rows see Table 4.

Tab. 4. Comparison between analytical solution and ANSYS

Horizontal level (m)	Coefficient of correlation (-)
1,5	0,999
2,5	1,000
3,5	1,000

Fig.13 shows relative deviation of analytical solution and ANSYS.

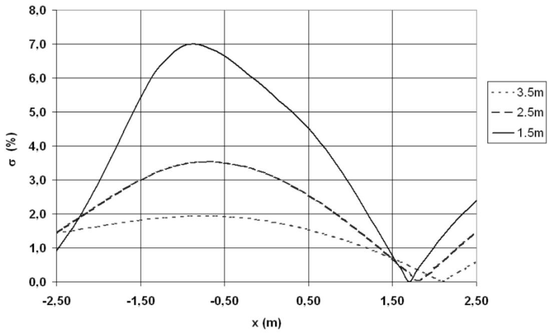

Fig. 13 Chart of relative deviation

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REFERENCES

[1] Majer, D.; Polák , J.: Metody řešení elektrických a magnetických polí, Praha 1983.

[2] Orwis, W.J.: Microsoft Excel pro vědce a inženýry, Praha 1996

[3] Paleček, J.: Vybrané kapitoly z problematiky elektrotechniky v dopravním inženýrství, Ostrava 1996.

[4] Štafl, M.: Elektrodynamika ve stavbě elektrických strajů, Praha 1962

[5] ANSYS Help System, rev. 9.0

[6] ANSYS LF Electromagnetic guide, rev. 9.0