NON DESTRUCTIVE TESTING - IDENTIFICATION OF DEFECTS IN MATERIALS

T. Bachorec, J. Dedkova

Brno University of Technology, Brno, Czech Republic, Kolejní 2906/4, 612 00 BRNO, CZECH REPUBLIC
e-mail: bachorec@feec.vutbr.cz

Summary In electrical impedance tomography (EIT) currents are applied through the electrodes attached on the surface of the object, and the resulting voltages are measured using the same or additional electrodes. Internal conductivity distribution is recalculated from the measured voltages and currents. The problem is very ill posed, and therefore, regularization has to be used. The aim is to reconstruct, as accurately as possible, the conductivity distribution σ in phantom using finite element method (FEM). In this paper are proposed variations of the regularization term, which are applied to non-destructive identification of defects (voids or cracks) in conductive material.

1. INTRODUCTION

EIT is a soft-field tomographic modality, where images of the electrical conductivity distribution in a volume can be reconstructed from voltage measurement captured on its boundaries. Usually, a set of voltage measurements is acquired from the boundaries of a conductive volume, whilst this is subjected to a sequence of low-frequency current patterns. In principle, measuring both the amplitude and the phase angle of the voltage can result in images of the electric conductivity and permitivity in the interior of a body. Alternating current patterns are preferred to DC to avoid polarization effects. In the usual frequency range (below 1 MHz) the field can be considered a steady current field, which is governed by the Laplace equation. The theoretical background of EIT is given in [1].

The forward EIT calculation yields an estimation of the electric potential field in the interior of the volume under some Neumann and Dirichlet boundary conditions. The Finite Element Method in two or three dimensions is exploited for the forward problem with current sources.

The EIT image reconstruction problem is an ill-posed inverse problem of finding such σ that minimizes some optimisation criterion. The optimization necessitates algorithms that impose regularization and some prior information constraint. The regularization techniques vary in their complexity. This paper proposes some new possibilities to be used for the acquirement of more accurate reconstruction results.

2. FORWARD SOLUTION

EIT is used to reconstruct the conductivity distribution by the measured surface electric potential distribution around the phantom when injecting current into the object. The electric field intensity is supposed to be on frequencies up to 1 MHz irrotational due to the low conductivity of a medium. The scalar potential U can be therefore introduced, so that the resulting field is conservative and the continuity equation for the volume current density can be expressed from the potential U

$$\mathrm{div}\left(\sigma\,\mathrm{grad}U\right) = 0\,. \qquad (1)$$

Equation (1) together with the modified complete electrode model equations [2] is discretized by the FEM in the usual way. Using FEM we calculate approximate values of electrode voltages for the approximate element conductivity vector σ (NE x 1), NE is the number of finite elements. In the following we assume only constant value of σ on each finite element.

3. INVERSE PROBLEM

Image reconstruction of EIT is an inverse problem, in which with the aid of known current patterns and corresponding measured voltages the volume conductivity distribution of the interior is estimated. In this paper we use a deterministic approach based on the Least Squares (LS) method. Due to ill-posed nature of the problem, regularization has to be used. The regularized solution is the solution of the following nonlinear minimization problem:

$$\min_\sigma \Psi(\sigma)\,,$$

here $\Psi(\sigma)$ is the suitable objective function.

A Generalized Tikhonov Regularization Method

At first the standard (General) Tikhonov Regularization Method (GTRM) described in [3], was used for the solution of this inverse EIT problem. So, we have to minimize the objective function $\Psi(\sigma)$

$$\Psi = \frac{1}{2}\sum \left\|\boldsymbol{U}_{\mathrm{MEAS}} - \boldsymbol{U}_{\mathrm{FEM}}\left(\sigma\right)\right\|^2 + \alpha\left\|\boldsymbol{R}\boldsymbol{\sigma}\right\|^2. \qquad (2)$$

Here, σ is the vector of iterated volume conductivities in 2D or 3D, respectively. $\boldsymbol{U}_{\mathrm{MEAS}}$ is the vector of nodal voltages, calculated from the phantom with the known volume conductivities, $\boldsymbol{U}_{\mathrm{FEM}}$ is the vector iteratively calculated by using the FEM applied to (1). Further, α is the regularization parameter, and $\boldsymbol{R}$ is a suitable regularization matrix, connecting adjacent elements of the different conductivity values.

For the solution of (2) we can used a Newton-type method and after the linearization we used the iteration procedure

$$\boldsymbol{\sigma}_{i+1} = \boldsymbol{\sigma}_i + (\boldsymbol{J}_i^T \boldsymbol{J}_i + \alpha \boldsymbol{R}^T \boldsymbol{R})^{-1} (\boldsymbol{J}_i^T (\boldsymbol{U}_{\mathrm{MEAS}} - \boldsymbol{U}_{\mathrm{FEM}}(\boldsymbol{\sigma}_i)) - \alpha \boldsymbol{R}^T \boldsymbol{R} \boldsymbol{\sigma}_i) \quad (3)$$

Here i is the i-th iteration and $\boldsymbol{J}$ is the Jacobian for forward operator $\boldsymbol{U}_{\mathrm{FEM}}$ and can be calculated very effectively, for example by using the reciprocity principle [4].

B Primal Dual Interior Point Method Algorithm

To recover conductivity distribution we used the widely known method Total Variation PD-IPM, too.

We again minimize the primal objective function $\Psi(\sigma)$

$$\Psi = \frac{1}{2}\sum \left(\boldsymbol{U}_{\mathrm{MEAS}} - \boldsymbol{U}_{\mathrm{FEM}}(\sigma)\right)^2 + \alpha \mathrm{TV}_\beta \quad (4)$$

where $\boldsymbol{U}_{\mathrm{FEM}}$ is the vector iteratively calculated using the FEM, $\boldsymbol{U}_{\mathrm{MEAS}}$ is the vector of nodal voltages calculated from the model with the known conductivities, α is the regularization parameter and

$$\mathrm{TV}_\beta = \sum_{all\,elements} \int |\mathrm{grad}\,\boldsymbol{\sigma}|\, d\Omega = \sum \sqrt{\|\boldsymbol{R\sigma}\|^2 + \beta}. \quad (5)$$

Here $\boldsymbol{R}$ is a suitable regularization matrix and β is a small positive parameter, which represents an influence on the smoothing of $\Psi(\sigma)$. To find homogenous σ we used the PD-IPM algorithm similar to that described in [5].

There is often very difficult to ensure the stability and the sufficient accuracy of the required solution in applying both described reconstruction algorithms, because they are very sensitive on the suitable choice of the regularization parameter α as well as of the initialize values (starting value) of conductivity σ.

The stability of the GTRM algorithm is less sensitive to the setting of the starting value of conductivity than the Total Variation PD-IPM algorithm, which is in mostly cases unstable. Based on the results of many numerical experiments, we can say that we obtain the higher accuracy of the reconstruction results for smaller value of the parameter α, but with the decreasing of this parameter the instability of the system is increasing.

4. NUMERICAL SIMULATIONS

To recover conductivity distributions was used LS method with different type of the regularization's way. Furthermore, we compare the results obtained by the GTRM, by the PD-IPM algorithm and by their combination with different values of the regularization parameter α during reconstruction process and of the initialize values of conductivity σ. To evaluate the quality of simulation results, the total error Err of the recovered conductivity distribution σ is defined as

$$Err = \sqrt{\frac{\sum_{i=1}^{NE} (\sigma(i) - \sigma_{orig}(i))^2}{\sum_{i=1}^{NE} (\sigma_{orig}(i))^2}}\; 100\ \% \quad (5)$$

Here σ_{orig} (in S/m) is the actual (original) value, σ is the value recovered by EIT.

The above proposed algorithms for 2D model have been implemented into the modification of program [6], which has been written in MATLAB 7.0. An example of 2D arrangement for a numerical experiment is given in Fig. 1. A model of an annular ring is shown with outer radius 10 cm and inner radius 7cm, the total number of electrodes is 20. We applied a total of 20 different cosine current excitations calculating 19 independent nodal voltages for each excitation.

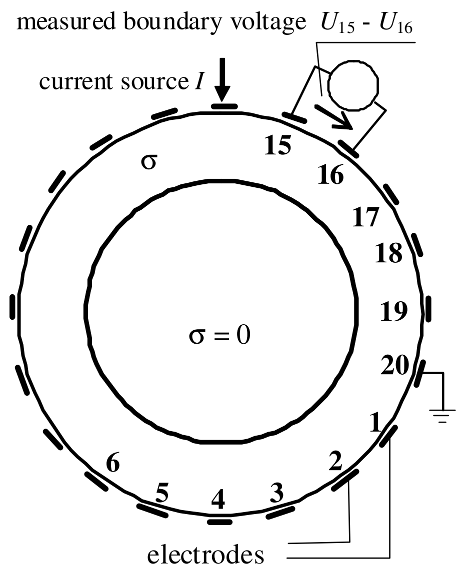


Fig. 1. Arrangement of 2D model

Example 1

In the Fig. 2 you can see the FEM mesh for the calculations of the gradients, voltage reference values, and the Jacobians during iterations. The total number of elements is 400; the number of nodes is 269. We assume a homogeneous object with conductivity 0.5 S/m on all elements except the chosen ones, where values of conductivity (on eight darkly marked elements in Fig. 2) are 0 S/m. These elements can represent some cracks or voids.

The example of a numerical experiment using the Tikhonov regularization is shown in Fig. 3. In Fig. 3a) the conductivity σ (in S/m) is the value on each of elements recovered by EIT after iterations process. The starting values of conductivity are 0.5 S/m on all elements; the starting value of parameter α is 5.10^{-5}.

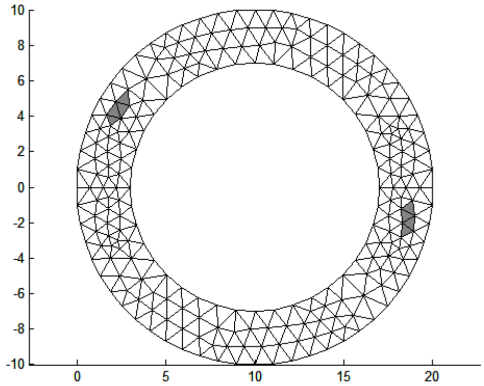
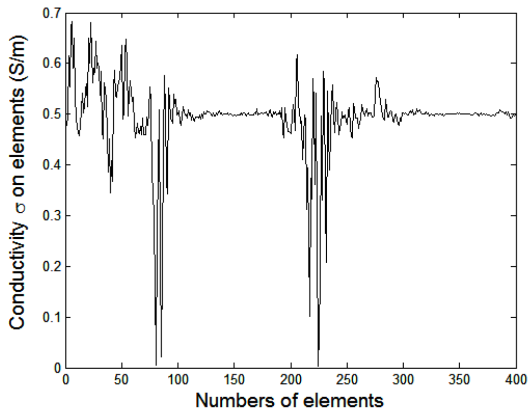


Fig. 2. FEM grid and regions with non-homogeneity

a)



b)

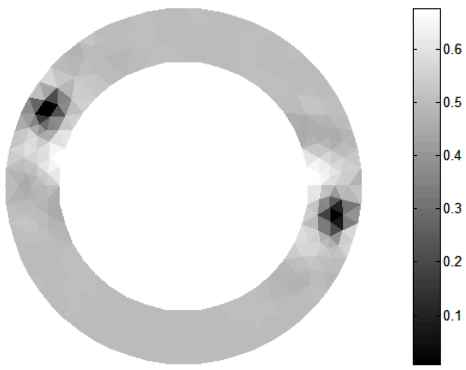
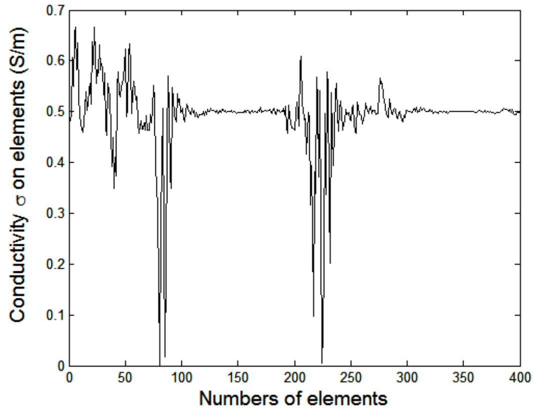


Fig. 3. GTRM algorithm

In Fig. 3b) are shown places of identified non homogeneity. The value of parameter α decreased during regularization process and this process becomes unstable for parameter α which is less than 2.10^{-13}, value of the objective function Ψ is $7.8\ 10^{-16}$ and total error *Err* is 11 %. The sufficient number of iterations for each parameter α is 5.

The similar results we obtain using the PD-IPM algorithm for regularization. The recovered value of conductivity σ on each of elements is shown in Fig. 4a). As in the previous example the starting values of conductivity are 0.5 S/m on all elements; the starting value of parameter α is 5.10^{-8}.

a)



b)

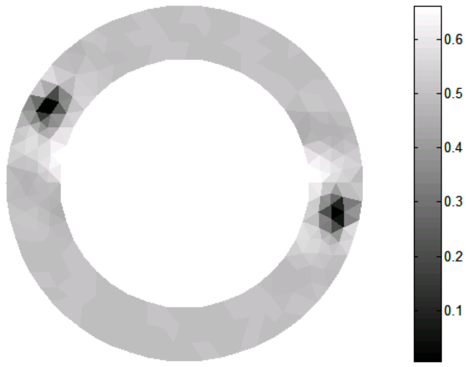


Fig. 4. TV PD-IPM algorithm

In Fig. 4b) are shown recovered places of identified non homogeneity. The value of parameter α decreased during regularization process and this process becomes unstable for parameter α which is less than 5.10^{-13}, value of the objective function Ψ is 0.08 and total error *Err* is 10.8 %. The sufficient number of iterations for each parameter α is 10.

Example 2

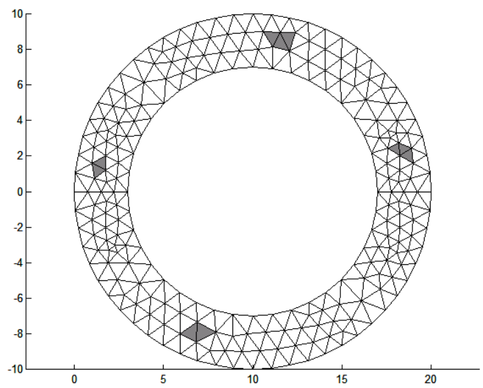


Fig. 5. FEM grid and regions with non-homogeneity

In the Fig. 5 you can see ten darkly marked elements on FEM grid, which can represent another arrangement of the cracks or voids. On these elements are values of conductivity 0 S/m.

The example of a numerical experiment using the Tikhonov regularization is shown in Fig. 6. In Fig. 6a) the conductivity σ (in S/m) is the value on each of elements recovered by EIT after iterations process. The starting values of conductivity are 0.5 S/m on all elements; the starting value of parameter α is 5.10^{-5}.

In Fig. 6b) are shown places of identified non-homogeneity. The value of parameter α decreased during regularization process and this process becomes unstable for parameter α which is less than 2.10^{-13}, value of the objective function Ψ is $6\ 10^{-15}$ and total error *Err* is 17 %. The sufficient number of iterations for each parameter α is 5.

a)

b)

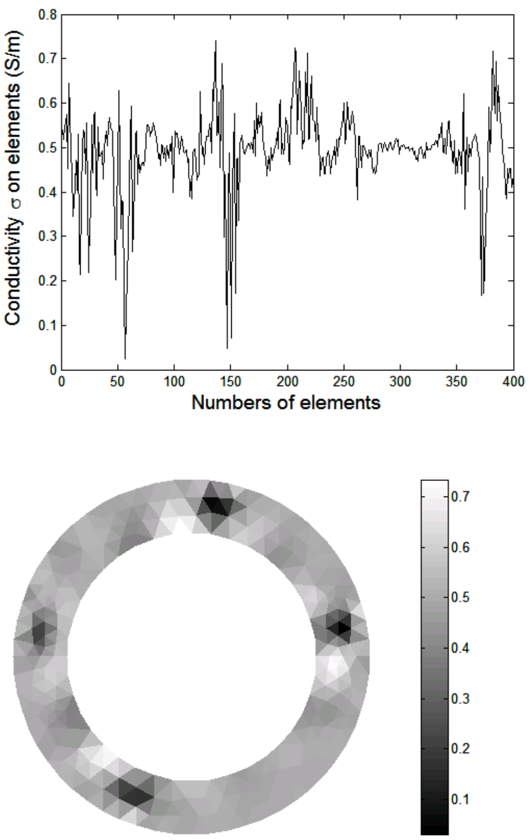


Fig. 6. GTRM algorithm

The similar results we obtain using the PD-IPM algorithm for regularization. As in the previous example the starting values of conductivity are 0.5 S/m on all elements; the starting value of parameter α is 5.10^{-8}. The value of parameter α decreased during regularization process and this process becomes unstable for parameter α which is less than 3.10^{-11}, value of the objective function Ψ is 0.2 and total error *Err* is 15 %. The sufficient number of iterations for each parameter α is 10.

5. CONCLUSION

We succeeded in implementing the above-mentioned regularization of GTRM and TV PD-IPM algorithm for EIT in the environment of EIDORS 2D program system [6]. After several series of computational experiments it was established that satisfactory results could be often obtained when we use for a reconstruction at first the GTRM and then the recovered values of conductivity we set to starting values for application of the TV PD-IPM algorithm.

It is shown that we can very successfully identify voids or cracks in conductive materials by using these algorithms. The proposed method is expected to be used for the non-destructive testing materials for 3D model, too. The algorithms for 3D model will be implemented into program written in ANSYS and we assume that the obtained reconstruction results will be in accordance with the results from Example 1 and Example 2.

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REFERENCES

[1] M. Cheney, D. Isaacson, and J.C.Newell: Electrical impedance tomography. SIAM Rev. Vol. 41, no. 1, 1999, pp.85-101.

[2] E. Somersalo, M. Cheney, D. Isaacson: Existence and uniqueness for electrode models for electric current computed tomography. SIAM J. Appl. Math. Vol. 52, 1992, pp.1023-1040.

[3] M. Vauhkonen, D. Vadász, P. A. Karjalainen, E. Somersalo, and J. P. Kaipio: Tikhonov regularization and prior information in electrical impedance tomography. IEEE Trans. Med. Eng., Vol. 17, 1998, pp.285-293.

[4] B. Brandstätter: Jacobian calculation for electrical impedance tomography based on the reciprocity principle. IEEE Transaction on Magnetics, Vol. 39 No. 3, pp.1309-1312, 2003.

[5] A. Borsic: Regularization methods for Imaging from Electrical Measurement. PhD. Thesis, Oxford Brookes University, 2002.

[6] M. Vauhkonen, W. R. B. Lionheart, L. M. Heikkinen, P. J. Vauhkonen, J. P. Kaipio: A MATLAB package for the EIDORS project to reconstruct two-dimensional EIT images. *Physiol. Meas.* Vol. 22, pp. 107-111, 2001.