Positioning the Adjacent Buried Objects Using UWB Technology Combine with Levenberg-Marquardt Algorithm

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1. Introduction

Recently, methods to determine the position of objects in Non-Light-Of-Sight (NLOS) transmission environments have attracted a lot of researches and achieved significant results in Internet of Things (IoT) networks, indoor positioning systems, hospital monitoring and surveillance systems, radar penetration, etc.

A number of signal analysis techniques and system models are proposed for distance measurement and object positioning such as Radio Frequency Identification in IoT network applications [1], using UWB technology combined with median and Kalman filters [2] in free space. In the underground environment, building structures, various techniques used for locating the underground pipelines such as Ground Penetrating Radar (GPR) [3] and [4]. The detection of buried objects in shallow sea with large bandwidth and low frequency electromagnetic waves proposed in [5] can overcome the limitations of acoustic waves and blue-green lasers.

From the above research results, it can be seen that the resolution and detection distance of these non-destructive techniques strongly depend on the signal shapes, the system bandwidth and the material of the buried objects (plastic or metal) as well as information of environmental properties (relative permittivity).

The GPR devices can provide a high image resolution and efficient data processing when using amplitude modulated signals [6], wide-band chaotic signals based on time domain correlation and back-projection algorithm [7], or combines GPR and the electric field...
method in detecting the depth and radius of plastic pipe [5], using fast and sophisticated reconstruction technique to create the 2D image of the buried target for GPR systems [9]. However in these techniques, the images of underground objects heavily depends on the propagation velocity in signal processing, hence the determination of the relative permittivity of the environment is necessary, but the relative permittivity cannot be determined by image processing.

Using the acoustic signal, it is possible to locate buried objects even when the transmission medium is wet, but this method is strongly influenced by noise [10] and also impossible to determine the wave velocity. Moreover, when the distance between the buried objects is very small, the methods based on GPR, machine learning and acoustic signals are indistinguishable. Therefore, the use of ultra-wide band radio impulses to increase discrimination of identifying adjacent buried objects is a good solution. The very large bandwidth and high spatial resolution makes IR-UWB signal attractive for penetration applications [11] and [12] and became one of the good candidates for positioning techniques especially in the building structures [13] and [14].

In order to increase the ability to distinguish adjacent buried objects and hence, improve the positioning errors, we propose the method of correlation function separation, called CFST combine with the Lervenberg-Marquardt Algorithm (LMA) [15] which is used to process the reflected UWB signal from the buried objects. The method can be used to determine the relative permittivity of the environment, and the positions of the buried objects for both homogeneous and heterogeneous environments. The contributions of the paper are summarized as follows:

- Constructing a system model for positioning a single buried object in a homogeneous medium, and determining the position of the object and the relative permittivity of environment by using the LMA.

- A new positioning algorithm for adjacent buried objects in the homogeneous environment is proposed, named CFST.

The rest of the paper is organized as follows. Section 2 presents the positioning of buried object system model. Mathematical analysis of the correlation functions for the cases of single and adjacent buried objects and LMA are presented in Subsec. 2.2 and Subsec. 2.3. Numerical results are presented in Sec. 3. Finally, Sec. 4 concludes the work.

2. Proposal of Positioning Algorithm

2.1. System Model

The model of a UWB penetrating system is illustrated in Fig. 1, the transmission medium has a relative permittivity of \( \varepsilon \), receiver and transmitter antennas are placed in the same position with an assumption of height of 0 m to the environment’s surface. The transmitted signal is UWB radio pulse and denoted as \( s(t) \), the reflected signal is denoted as \( r(t) \), which is a sum of the reflected signals from the buried objects. To determine the propagation distance, on the receiver side, \( r(t) \) is correlated with the template signal, then the delay time between the transmitted and reflected signals is computed. The delay time in this paper is called as the traveling time. The parameters of the model are described as follows:

- The transmitted IR-UWB signal takes the form [15]:

\[
s(t) = \sqrt{P} \sum_{i=0}^{N} p(t - iT),
\]

where \( P \) is the transmit power, \( N \) is the number of transmitted pulses, \( T \) is the repetitive period of the pulse and \( p(t) \) is the signal pulse for IR-UWB systems, including Gaussian monocycles, Manchester monocyte and modified Hermite pulse [17].

- The reflected signal is described as:

\[
r(t) = \sum_{i=0}^{M} [A_i s(t - \tau_i) + n_i(t)],
\]

where \( M \) is the number of buried objects, \( \{A_i\}_{i=1}^{M} \) represent the attenuation of the transmission medium and distance, \( \tau_i \) represents the traveling time of the reflected signals from the \( r \)th buried object, and \( n_i(t) \) is additive white Gaussian noise.

![Fig. 1: The model of a UWB penetrating system.](image-url)
• The template signal at the receiver is \( p(t) \), and the correlation function at the receiver side is determined by:

\[
R(\delta) = \int_{-\infty}^{\infty} r(t)p(t-\delta)dt. \tag{3}
\]

• The traveling time is calculated as:

\[
\tau_i = \text{Arg} \max \{ R(\delta) \} = \text{Arg} \max_{\delta} \left\{ \int_{-\infty}^{\infty} r(t)p(t-\delta)dt \right\}. \tag{4}
\]

• The propagation velocity in the system is \([18]\):

\[
V = \frac{c}{\sqrt{\varepsilon}}. \tag{5}
\]

where \( c = 3 \cdot 10^8 \text{ m s}^{-1} \) is the velocity of light in the vacuum environment.

• The distance from the device to the \( i \)-th buried object with assumption that the transmission medium is homogeneous:

\[
l_i = \frac{1}{2} V \tau_i. \tag{6}
\]

In this model, we consider two cases, in the first case, two buried objects are assumed to be very close to each other (T1, T2), in which, the reflected signals from two buried objects overlap. In the other case, two buried objects are far away from each other (T3, T4), in which, the reflected signals from those objects do not overlap. An example illustrates two types of the reflected signal as shown in Fig. 2.

![Fig. 2: The transmitted and reflected signals with added noise.](image)

In the UWB systems, the pulse shape has a strong influence on their performance, we first restrict our analysis to the 2nd Gaussian monocycle, the 3rd and 4th order are investigated in Subsec. 3.2 for comparing and choosing the suitable pulse shape. The 2nd order Gaussian monocycle given by \([17]\):

\[
g_2(t) = B_{2p} \frac{d^2}{dt^2} e^{-2\pi \left( \frac{t}{T_p} \right)^2} = \left[ 1 - 4\pi \left( \frac{t}{T_p} \right)^2 \right] e^{-2\pi \left( \frac{t}{T_p} \right)^2}, \tag{7}
\]

where \( T_p \) represents time normalization factor, the shapes of the auto-correlation functions of the 2nd, 3rd and 4th order Gaussian monocycles are presented in Fig. 3. The performance analysis of the system is considered first with a single buried object, then with adjacent buried objects in the homogeneous environment.

![Fig. 3: The auto-correlation function of the Gaussian pulses as a function of time.](image)

2.2. Positioning a Single Buried Object

As seen in Fig. 1, the parameters \( d_{ob}, Z_{ob} \) are depth and horizontal coordinates of a single buried object in the 2D space; \( Z_{Dei}, \Delta Z, \) and \( l_i \) are horizontal position, the movement step of the device and distance from the object to device at its \( i \)-th position, respectively. These parameters are determined as follows:

\[
Z_{Dei} = i \cdot \Delta Z, \quad l_i = \sqrt{(Z_{ob} - Z_{Dei})^2 + d_{ob}^2}. \tag{8}
\]

The traveling time is given by:

\[
\tau_i = 2 \sqrt{\frac{\varepsilon \left[ (Z_{ob} - Z_{Dei})^2 + d_{ob}^2 \right]}{c}} = \frac{2 \sqrt{\varepsilon \left[ (Z_{ob} - i \cdot \Delta Z)^2 + d_{ob}^2 \right]}}{c}, \tag{9}
\]

\( \tau_i \) is calculated according to Eq. (1). In Eq. (9), the values of \( \tau_i, i, \text{ and } Z_{Dei} \) are known, and the values of \( \varepsilon, Z_{ob}, \text{ and } d_{ob} \) are estimated by the LMA so that the cost function reaches the minimum value. Accordingly, those parameters are considered as the coefficients of the nonlinear equations. Their values are determined based on a given set of pair \((Z_{Dei}, \tau_i)\), and the cost function is given in Eq. (11).
The unknown parameter vector is denoted by:
\[ \vec{X} = (\varepsilon, Z_{ob}, d_{ob}) \]  
where \( J \) is the Jacobian matrix, whose \( i^{th} \) row equals \( J_i \) and \( \vec{f} (Z_{De}, \vec{X}) \), \( \vec{\tau} \) are vectors of \( i^{th} \) component \( f (Z_{De}, \vec{X}) \) and \( \tau_i \) respectively, we have:
\[ J_i = \frac{\partial f (Z_{De}, \vec{X})}{\partial \vec{X}}. \]  

The sum \( \bar{S} (\vec{X} + \vec{\theta}) \) has its minimum at zero gradient with respect to \( \vec{\theta} \), hence \( \vec{\theta} \) can be determined satisfying:
\[ [J^T J + \lambda \text{diag} (J^T J)] \vec{\theta} = J^T \left[ \vec{\tau} - \vec{\bar{f}} (Z_{De}, \vec{X}) \right], \]  
where the damping factor \( \lambda \) (non-negative) is adjusted at each iteration. If \( \bar{S} \) is reduced rapidly, a smaller value of \( \lambda \) can be used, whereas if \( \bar{S} \) does not reduce, \( \lambda \) can be increased.

**Step 3:** The updated step vector is computed as follows:
\[ \vec{\theta} = [J^T J + \lambda \text{diag} (J^T J)]^{-1} J^T \left[ \vec{\tau} - \vec{\bar{f}} (Z_{De}, \vec{X}) \right]. \]  

The algorithm repeats steps 2 and 3 until the constraint condition in Eq. (11) is satisfied. And the outputs of LMA are the final estimated values of system parameters \( \vec{X} = (\varepsilon, Z_{ob}, d_{ob}) \).

### 2.3. Positioning the Adjacent Buried Objects

Considering the system model in Fig. 4 with two buried objects, the received signal is the sum of the reflected signals from these objects, and are denoted by \( r_1(t) \) and \( r_2(t) \). In case, the buried objects are very far from each other, such as T3 and T4 in Fig. 4, the reflected signals \( r_1(t) \) and \( r_2(t) \) do not overlap, consequently we have applied the procedure to locate single object as presented in Subsec. 2.2. However, in case, the buried objects are near each other, such as T1 and T2 in Fig. 4, the reflected signals \( r_1(t) \) and \( r_2(t) \) overlap. In this scenario, as shown in Fig. 5, the correlation function shapes are changed with different cases of the buried objects: there is one buried object, two adjacent buried objects and two apart buried objects with \( Z_{mov} \) is the distance between the buried objects.

In the case of two apart buried objects, it is entirely possible to apply the method in Subsec. 2.2 to locate one by one buried object. In the case of two adjacent buried objects, the proposed CFST is applied to determine the second object by using the correlation function values and the position of the first object. The
correlation function has the form:

\[ R(\delta) = \int_{-\infty}^{\infty} r_\Sigma(t)p(t-\delta)dt = \int_{-\infty}^{\infty} [r_1(t)+r_2(t)]p(t-\delta)dt = R_1(\delta) + R_2(\delta), \]

and the traveling time can be calculated as (Total):

\[ \tau_\Sigma = \text{Arg max}_\delta \{ R(\delta) \}. \]

Using the received signal \( r_\Sigma(t) \), the traveling time from the second object to the device is denote by \( \tau_2 \) and can be computed by CFST with two expansions. Firstly, by Subtracting the Correlation Functions (SCF):

\[ \tau_2 = \delta_{\text{op}} = \left[ \text{Arg max}_\delta \{ R(\delta) - R_1(\delta) \} \right]. \]

Secondly, by Dividing the Correlation Functions (DCF):

\[ \tau_2 = \delta_{\text{op}} = \left[ \text{Arg max}_\delta \left\{ \frac{R(\delta)}{R_1(\delta)} \right\} \right]. \]

Fig. 5 illustrates the shapes of the correlation functions of reflected signals with the template signal in the case of two adjacent buried objects in the homogeneous environment. The traveling time \( \tau_2 \) can be estimated according to SCF, DCF and the position of the second object can be located in the same way as presented in the Subsec. 2.2. by using the LMA in which \( \tau \) is replaced by \( \tau_2 \). The flowchart of the proposed CFST is illustrated in the Fig. 7.

3. Numerical Results and Comparisons

3.1. System Parameters

An example of the parameters of a penetrating UWB system is listed in Tab. I. Our above analysis presents that the adjacent buried objects can be located one by one, the position of the following object is determined based on the data set of the correlation function value of the previous object. All the numerical results in this paper were computed using Matlab, the data set of the reflected signals were generated by simulation. Furthermore, the localization technique is based...
on the Time Of Arrival (TOA), and the Root Mean Squared Error (RMSE) of the CFST is determined by the Eq. (21) and compared with the Cramer-Rao Lower Bound (CRLB) in Eq. (22).

### Tab. 1: Simulation parameters [21].

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Notation</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Impulse width</td>
<td>PW</td>
<td>0.7 ns</td>
</tr>
<tr>
<td>Transmitted power</td>
<td>$P_T$</td>
<td>-5.4 dBm</td>
</tr>
<tr>
<td>Noise power</td>
<td>$N_0/2$</td>
<td>-77 dBm</td>
</tr>
<tr>
<td>Time normalization factor</td>
<td>$T_p$</td>
<td>0.2877 ns</td>
</tr>
<tr>
<td>Effective bandwidth</td>
<td>$\Delta F$</td>
<td>3.5 GHz</td>
</tr>
<tr>
<td>Relative permittivity</td>
<td>$\varepsilon$</td>
<td>4.5</td>
</tr>
<tr>
<td>The repetitive period</td>
<td>$T$</td>
<td>50 ns</td>
</tr>
<tr>
<td>Number of pulse</td>
<td>$N$</td>
<td>100</td>
</tr>
<tr>
<td>Movement step of the device</td>
<td>$\Delta Z$</td>
<td>1 cm</td>
</tr>
</tbody>
</table>

The Root Mean Squared Error (RMSE) of the CFST in determining the traveling time $\tau_2$:

$$\text{RMSE} = \sqrt{\frac{1}{N} \sum_{k=1}^{N} (\hat{\tau}_k - \tau_2)^2}. \quad (21)$$

The CRLB [20] on the standard deviation of an unbiased TOA estimator $\hat{\tau}$ is given by:

$$\sqrt{\text{Var}(\hat{\tau})} \geq \frac{1}{2\sqrt{2\pi \text{SNR} \Delta F}}, \quad (22)$$

where SNR, $\Delta F$ are signal to noise ratio and effective bandwidth, respectively. The change of RMSE vs. SNR for SCF, DCF and Total methods in estimating the traveling time $\tau_2$ are illustrated in Fig. 8 with the case of two adjacent buried objects in the homogeneous environment, the first buried object at a depth of 30 cm, and the second buried object is 4 cm away from the first one.

![Fig. 8: RMSE vs. SRN for CFST method and CRLB.](image)

### 3.2. Positioning Single Buried Object

Fig. 9 shows the result of determining the position of a single buried object using different types of pulses with the same parameter values listed in Tab. 1. To illustrate the graph more clearly, the units of $Z_{ob}$ and $d_{ob}$ in the figures are set to meters.

![Fig. 9: The traveling time change according to the position of device (top) and the estimated positions of the single buried object (bottom).](image)

The dashed black line denotes the traveling time according to the position of the device (see Eq. (4) and Eq. (6)) with the true values of the buried object ($Z_{ob}$, $d_{ob}$, $\varepsilon$) = (60 cm, 50 cm, 4.5) and the remaining lines represent values estimated by the LMA with the 2nd, 3rd and 4th order Gaussian monocycles. In addition, the true and estimated locations by the proposed method are indicated in Tab. 2, the positioning error using the 4th order pulses in this scenario has an average value of about 2.4 cm, of the 2nd order pulses is 4.5 cm, and of the 3rd order pulses is 6 cm.

### Tab. 2: The numerical results.

<table>
<thead>
<tr>
<th>Notation</th>
<th>$Z_{ob}$ (cm)</th>
<th>$d_{ob}$ (cm)</th>
<th>$\varepsilon$</th>
</tr>
</thead>
<tbody>
<tr>
<td>The true value</td>
<td>60</td>
<td>50</td>
<td>4.5</td>
</tr>
<tr>
<td>The estimated values with 2nd order Gaussian</td>
<td>64.12</td>
<td>46.05</td>
<td>4.2809</td>
</tr>
<tr>
<td>The estimated values with 3rd order Gaussian</td>
<td>66.12</td>
<td>44.77</td>
<td>4.1323</td>
</tr>
<tr>
<td>The estimated values with 4th order Gaussian</td>
<td>61.18</td>
<td>47.63</td>
<td>4.3266</td>
</tr>
</tbody>
</table>

One can observe that the performance using the 4th order Gaussian monocycle is better than the others. This reason can be explained by comparing the auto-correlation functions of Gaussian monocycles as shown in Fig. 3. The shape of the auto-correlation function of the 3rd order Gaussian monocycle has maximum points at $\tau = 0.32$ ns and $\tau = 0$ ns while the 2nd and 4th order Gaussian monocycles have only one extreme point at $\tau = 0$ ns. This leads to determining the traveling time using the correlation function of the third-order Gaussian monocycle with larger error than using other monocycles.
3.3. Positioning Adjacent Buried Objects

Based on the sample set of the correlation values in the case of positioning single buried object, the results of locating two adjacent objects is illustrated in Fig. 10 with the distance between them being 2 cm.

As seen in Fig. 10, the estimated error of using the SCF is smaller than using the DCF. The results can be explained by observing Fig. 6, Fig. 8, Eq. (19) (SCF) and Eq. (20) (DCF), we can see that the error of estimating traveling time parameter depends on the values of the correlation function, SNR and these values are proportional to the amplitude of received signal, hence the estimation error depends on the amplitude of received signal. Moreover, comparing the SCF and the DCF, we observe that in DCF, the amplitude of the reflected signal from the second object is reduced by the the division of the correlation functions. Hence, the value of the correlation function of the DCF also reduces, causing higher errors when determining traveling time rather than using the SCF.

Fig. 10: Two adjacent buried objects: the traveling time change according to the device’s translation (top) and the estimated position of the object by the CFST (bottom).

Fig. 11: The true and estimated locations by CFST method.

GPR images (Multiresolution Monogenic Signal Analysis, Wide-band chaotic) are indistinguishable the adjacent and tangent hyperbolas, and greatly affected by background noise. Meanwhile, the CFST can be used to solve this problem. With the ability to separate the adjacent hyperbolas, the performance of CFST is better than in the conventional method.

Tab. 3: The comparison of different methods.

<table>
<thead>
<tr>
<th>Method</th>
<th>Distance error (cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>The Multiresolution Monogenic Signal Analysis [6]</td>
<td>5.8</td>
</tr>
<tr>
<td>Wideband chaotic [7]</td>
<td>10</td>
</tr>
<tr>
<td>CFST</td>
<td>3.52</td>
</tr>
</tbody>
</table>

4. Conclusions

In this paper, we propose a method to locate multi-buried objects, especially adjacent objects based on analysis of correlation function and LMA for IR-UWB penetrating systems, called CFST. Our analysis indicates that, by processing the values of the correlation function between the reflected and template signals, we can determine the characteristics of the environment and the location of buried objects. The proposed method can be applied to locate the single buried object and adjacent buried objects. The performance of the IR-UWB system is assessed based on positioning errors and the CRLB of the TOA method. These errors depends on the order of Gaussian monocytes and the method analyze correlation function. Hence, the selection of the order of Gaussian monocytes and the correlation function analysis method depends on the specific application.
Author Contributions

H.N. and H.D. developed the theoretical about UWB signal and correlation function processing, performed analytical calculations and simulations on Matlab. All authors contributed to the final version of the manuscript. H.P. supervises the entire content of the article.

References


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