CAPACITIVE COUPLING IN DOUBLE-CIRCUIT TRANSMISSION LINES

Zdeňka Benešová, Lenka Šroubová

University of West Bohemia Pilsen
Faculty of Electrical Engineering
Department of Theory of Electrical Engineering
Sady Pětatřicátníků 14, 306 14 Plzeň, Czech Republic

Summary The paper describes an algorithm for calculation of capacitances and charges on conductors in systems with earth wires and in double-circuit overhead lines with respect to phase arrangement. A balanced voltage system is considered. A suitable transposition of individual conductors enables to reduce the electric and magnetic fields in vicinity of overhead lines and to limit the inductive and capacitive linkage. The procedure is illustrated on examples the results of which lead to particular recommendations for designers.

1. INTRODUCTION

Overhead lines (HV and EHV) for high power transmission are often realised as double-circuit lines. Due to the geometrical small distance of parallel lines there may occur relevant inductive and or capacitive couplings either among the conductors of one line or among the conductors of both lines. To reduce the influence of capacitive and inductive linkage and their dependence on geometrical non-symmetry these lines are usually designed as transposed. However respecting concrete phase arrangement of conductors of the lines the transposition can have different efficiency. Via suitable distribution of the phases on each conductor one can reach a reduction of electrical and magnetic field near the ground [2], [5] and so to reduce negative impact of lines on the surroundings. Also there is possible to reduce inductive voltage drops of the line [1], [3], [4].

This paper describes an algorithm for calculation of charges of conductors and of capacitive linkage in systems with neutral conductors respecting the geometrical non-symmetry of the lines. An influence of distribution of the phases on magnitude and non-symmetry of the linkage is investigated also for the case of sinusoidal steady state and a balanced voltage system.

2. CAPACITIVE COUPLING IN M-PHASES SYSTEMS WITHOUT NEUTRAL CONDUCTORS

It is well known that relations among voltages and charges of the conductors (see Fig.1) can be described using potential coefficients α_{ij} and capacitance ones β_{ij}

(1) $$\underline{\mathbf{U}}_0 = \mathbf{A}\underline{\mathbf{q}}$$

where $\underline{\mathbf{U}}_0^{\mathrm{T}} = [U_{10}, U_{20} \ldots U_{M0}]$, U_{i0} is a phasor of a phase-to-neutral voltage of conductor i, $\underline{\mathbf{q}}^{\mathrm{T}} = [q_1, \ldots, q_M]$, q_i is a phasor of a charge of conductor i, $\mathbf{A}\,(M,M) = \{\alpha_{ij}\}$ is a matrix of potential coefficients depending on a geometrical configuration of the lines

(2) $$\alpha_{ii} = \frac{1}{2\pi\varepsilon_0 l}\ln\frac{2h_i}{R_i} \qquad \alpha_{ij} = \frac{1}{2\pi\varepsilon_0 l}\ln\frac{b_{ij}}{d_{ij}}$$

b_{ij} is a distance between conductor i and the image of conductor j (the method of images), d_{ij} is a distance between conductors i and j. From the matrix of capacitance coefficients

(3) $$\mathbf{B} = \{\beta_{ij}\} = \mathbf{A}^{-1}$$

we can determine partial capacitances among the phase conductor C_{ij} and capacitances of conductors against the ground C_{ii} (see Fig.2) from the formulae

(4) $$C_{ij} = -\beta_{ij} \qquad C_{ii} = \sum_{j=1}^{M}\beta_{ij}$$

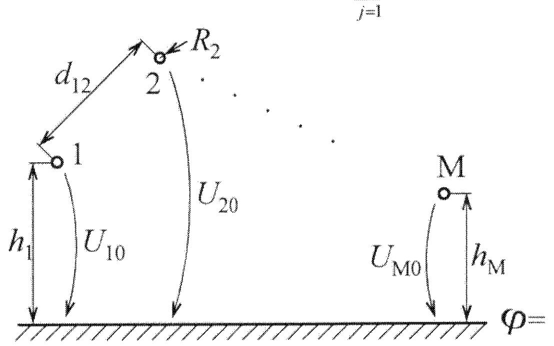

Fig. 1. Conductors of M-phases system.

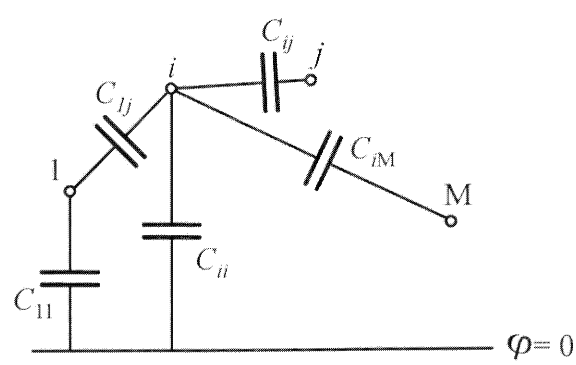

Fig. 2. Partial capacitances of M-phases system.

For the three-phase systems there is usually defined an effective capacitance between the conductor and the ground respecting as well as self as mutual capacitances. A formulae usually used in power engineering is derived on an assumption of geometrical symmetry of lines. Mutual capacitances in delta connection are transposed into star connection with assumption that star point is grounded.

Than effective capacitance of the phase conductor against ground is

(5) $$C_e = C_{ii} + 3C_{ij}$$

The effective capacitance C_e is used as for solution of fault conditions as for determining so called charging (capacitance) current caused by connection of an unloaded line to a voltage source. This current and the total reactive power supplied by capacitances is given for phase-to-neutral voltage U accordingly to equations

(6) $\boldsymbol{I} = j\omega C_e \boldsymbol{U} \qquad Q = 3\omega C_e U^2$

3. M-PHASE SYSTEM WITH NEUTRAL CONDUCTORS

Let us assume a system of M phase conductors with N neutral conductors (see Fig. 3). In the similar way to eq. (1) we can describe a relation among phase-to-neutral voltages and charges of all conductors respecting the fact that $U_i = 0$ for i = M+1, ..., M+N (neutral conductors are grounded) by matrix equation

$$(7) \qquad \begin{bmatrix} \underline{\mathbf{U}_M} \\ 0 \end{bmatrix} = \begin{bmatrix} \mathbf{A}_{MM} & \mathbf{A}_{MN} \\ \mathbf{A}_{NM} & \mathbf{A}_{NN} \end{bmatrix} \begin{bmatrix} \underline{\mathbf{q}_M} \\ \underline{\mathbf{q}_N} \end{bmatrix}$$

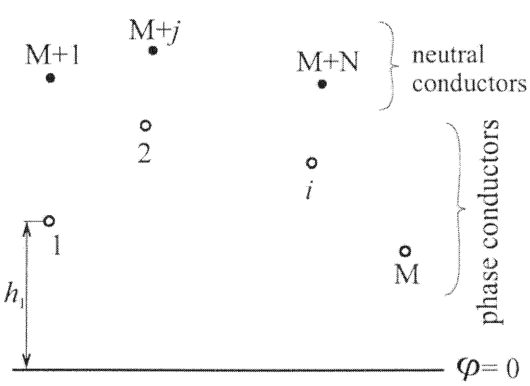

Fig. 3. The system of M phase conductors with N neutral conductors.

From the second row of the eq. (7) we can obtain the charges of neutral conductors

$$(8) \qquad \mathbf{A}_{NM}\underline{\mathbf{q}_M} + \mathbf{A}_{NN}\underline{\mathbf{q}_N} = 0$$

$$\Rightarrow \quad \underline{\mathbf{q}_N} = -\mathbf{A}_{NN}^{-1}\mathbf{A}_{NM}\underline{\mathbf{q}_M}$$

after substitution into eq. (7) we will get a relation among voltages and charges of phase conductors

$$(9) \qquad \underline{\mathbf{U}_M} = \left(\mathbf{A}_{MM} - \mathbf{A}_{MN}\mathbf{A}_{NN}^{-1}\mathbf{A}_{NM}\right)\underline{\mathbf{q}_M} .$$

Now a matrix of capacitance coefficients is defined by following equation

$$(10) \qquad \mathbf{B} = \left(\mathbf{A}_{MM} - \mathbf{A}_{MN}\mathbf{A}_{NN}^{-1}\mathbf{A}_{NM}\right)^{-1}$$

a matrix of charges of phase conductors will have the form

$$(11) \qquad \underline{\mathbf{q}_M} = \mathbf{B}\underline{\mathbf{U}_M} .$$

The following relation can determine the charges of neutral conductors

$$(12) \qquad \underline{\mathbf{q}_N} = -\mathbf{A}_{NN}^{-1}\mathbf{A}_{NM}\mathbf{B}\underline{\mathbf{U}_M} .$$

In accordance with relations (4) we can from eq. (10) define partial capacitances of phase conductors; by this way we get a set of capacitances corresponding with Fig. 2. The obtained capacitances C_{ii} however respect not only the capacitances against earth but also the linkage of phase and neutral conductors.

Example: For the three-phase system (see Fig. 4a for proportions of tower, phase conductors are signed as 1, 2, 3, neutral conductor as 0) determine the effective capacitance C_e, charging current and reactive power supplied by capacitances. The length of the line is ℓ = 10 km and the phase-to-neutral voltage U = 110 kV. We will perform the calculation by both ways

a) without respect to the capacitive coupling to the neutral conductor
b) with respect to the capacitive coupling C_{10}, C_{20}, C_{30} (see Fig. 4b)

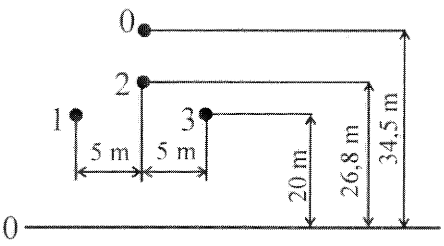

Fig. 4a. Example – arrangement of conductors.

Solution:

case a)

we will perform the calculation using eq. (2) – (6), and substituting into eq. (5)

$$(13) \quad C_{ii} = \frac{1}{3}\left(C_{11} + C_{22} + C_{33}\right), \quad C_{ij} = \frac{1}{3}\left(C_{12} + C_{13} + C_{23}\right)$$

we partially respect the geometrical non-symmetry of the line.

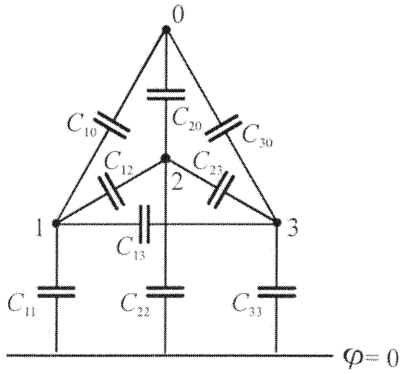

Fig. 4b. Partial capacitances in the three-phase system with the neutral conductor.

case b)

we will perform the calculation using the equations (2), (10), (4), (6), and effective capacitance of each phase conductor we calculate as follows

$$(14) \qquad C_{ei} = C_{ii} + C_{ij} + C_{ik} + \frac{C_{ij}C_{ik}}{C_{jk}} ,$$

$$i = 1,2,3; \quad j = 2,3,1; \quad k = 3,1,2 .$$

The charging current of the conductor i and the total reactive power are determined accordingly to eq. (6); these results are summarised in Tab. 1.

$$(15) \qquad \boldsymbol{I}_i = j\omega C_{ei}\boldsymbol{U}_i \quad , \qquad Q = \sum_{i=1}^{3} Q_i = \omega U^2 \sum_{i=1}^{3} C_{ei} .$$

Tab. 1.

ℓ= 10 km	C_{ii} [nF]	C_{ij} [nF]	C_e [nF]	I [A]	Q [kVAr]
ad a)	49,2	12,3			
3 conductors	45,0	9.6	82,1	2,84	936
	49,2	12,3			
ad b)	51,8	11,0	80,8	2,79	
4 conductors	50,4	9,0	85,9	2,97	941
	51,8	11,0	80,8	2,79	

4. CAPACITIVE COUPLING IN DOUBLE-CIRCUIT TRANSMISSION LINES

In the paper [1] we did show that a transposition of conductors of the double-circuit lines cannot be effective equally in all cases as a consequence of a geometrical non-symmetry of an inductive linkage. Using the algorithm which was derived above we will now analyse the capacitive linkage of double-circuit lines – see Fig.5. The phase conductors of line K are signed 1, 2, 3 and ones of line L are signed by 4, 5, 6, the neutral conductors are signed as 7, 8.

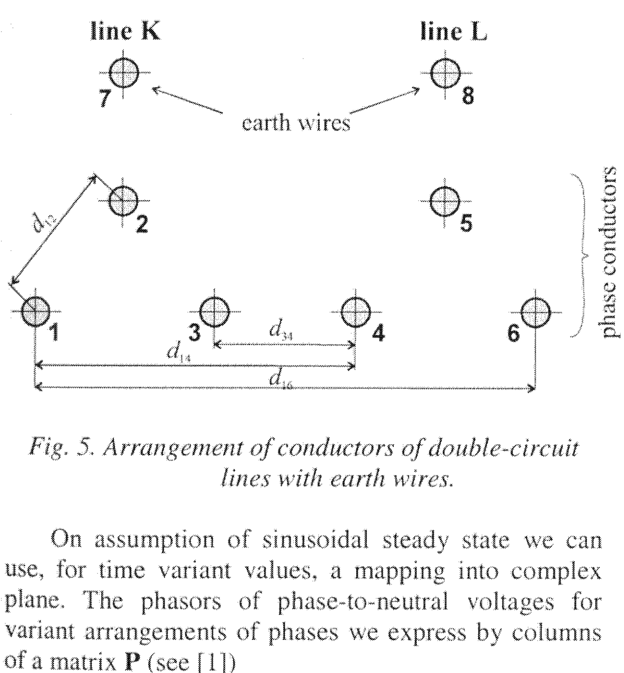

Fig. 5. Arrangement of conductors of double-circuit lines with earth wires.

On assumption of sinusoidal steady state we can use, for time variant values, a mapping into complex plane. The phasors of phase-to-neutral voltages for variant arrangements of phases we express by columns of a matrix $\underline{\mathbf{P}}$ (see [1])

$$\underline{\mathbf{P}} = \left[\underline{\mathbf{p}}_1, \underline{\mathbf{p}}_2, \dots \underline{\mathbf{p}}_6\right] =$$

$$(16) \qquad = \begin{bmatrix} 1 & 1 & a & a^2 & a^2 & a \\ a^2 & a & 1 & 1 & a & a^2 \\ a & a^2 & a^2 & a & 1 & 1 \end{bmatrix} .$$

Similarly to eq. (11) we will determine charges of phase conductors of lines K and L

$$(17) \qquad \underline{\mathbf{q}}_{\mathbf{M}} = \begin{bmatrix} \underline{\mathbf{q}}_{\mathbf{K}} \\ \underline{\mathbf{q}}_{\mathbf{L}} \end{bmatrix} = \begin{bmatrix} \mathbf{B}_{\mathrm{KK}} & \mathbf{B}_{\mathrm{KL}} \\ \mathbf{B}_{\mathrm{LK}} & \mathbf{B}_{\mathrm{LL}} \end{bmatrix} \begin{bmatrix} \underline{\mathbf{U}}_{\mathbf{K}} \\ \underline{\mathbf{U}}_{\mathbf{L}} \end{bmatrix} = = U \begin{bmatrix} \mathbf{B}_{\mathrm{KK}} & \mathbf{B}_{\mathrm{KL}} \\ \mathbf{B}_{\mathrm{LK}} & \mathbf{B}_{\mathrm{LL}} \end{bmatrix} \begin{bmatrix} \underline{\mathbf{p}}_{\mathbf{K}} \\ \underline{\mathbf{p}}_{\mathbf{L}} \end{bmatrix}, \qquad \mathrm{M} = 6$$

submatrices $\mathbf{B}_{\mathrm{KK}}$ a $\mathbf{B}_{\mathrm{LL}}$ express the relation between charge and phase-to-neutral voltage on lines K, L, resp., submatrices $\mathbf{B}_{\mathrm{KL}}$a $\mathbf{B}_{\mathrm{LK}}$ describe capacitive coupling between lines K and L. Then we can determine charging currents from the formula

$$(18) \qquad \begin{bmatrix} \underline{\mathbf{I}}_{\mathrm{K}} \\ \underline{\mathbf{I}}_{\mathrm{L}} \end{bmatrix} = j\omega \begin{bmatrix} \underline{\mathbf{q}}_{\mathrm{K}} \\ \underline{\mathbf{q}}_{\mathrm{L}} \end{bmatrix} .$$

As a consequence of the geometrical non-symmetry there is a capacitive coupling non-symmetry and therefore for a balanced voltage system is the set of currents and charges unbalanced, which we can express by formula

$$(19a) \qquad q_1 + q_2 + q_3 \neq 0 ; \qquad q_4 + q_5 + q_6 \neq 0$$

If we will denote

$$(19b) \qquad q_{C1} = q_1 + q_2 + q_3 \quad ; \qquad q_{C2} = q_4 + q_5 + q_6$$

then charges induced in the neutral conductors and on the ground surface will be q_i = q_{C1} + q_{C2}. The charges on the neutral conductors can be evaluated from the eq. (12), their value is proportional to non-symmetry of the capacitive linkage and can be used to determine optimal conductors transposition. Values of the charges of neutral conductors are listed for six basic arrangements according to eq. (16) in tab. 2a for the case of the tower Soudek and in tab. 2b for the case of vertical conductors arrangement. The phases are denoted by general symbols „•" , „°" and „x". The total inductance of the neutral conductors and values of electric field strength in height 1,2 m above the ground are also listed in these tables – they were calculated in the paper [3]. From the comparison with these results follows that optimal transposition of conductors in double-circuit lines is also minimizing not only non-symmetry of inductive and capacitive linkage but also the electric field strength E in height 1,2 m above the ground.

The total reactive power supplied by capacitances on line K is

$$(20) \qquad Q_{\mathrm{K}} = \mathrm{Im}\left\{\underline{\mathbf{U}}_{\mathbf{K}}^{+} \underline{\mathbf{I}}_{\mathbf{K}}\right\} = = \omega U^2 \, \mathrm{Im}\left\{\underline{\mathbf{p}}_{\mathbf{K}}^{+}\left(\mathbf{B}_{\mathrm{KK}} \underline{\mathbf{p}}_{\mathbf{K}} + \mathbf{B}_{\mathrm{KL}} \underline{\mathbf{p}}_{\mathbf{L}}\right)\right\}.$$

where the symbol $\mathbf{U}^+$ denotes a matrix associated with the matrix $\mathbf{U}$, the matrix with complex conjugated elements. The reactive power supplied on line L is similarly

$$(21) \qquad Q_{\mathrm{L}} = \mathrm{Im}\left\{\underline{\mathbf{U}}_{\mathbf{L}}^{+} \underline{\mathbf{I}}_{\mathbf{L}}\right\} = = \omega U^2 \, \mathrm{Im}\left\{\underline{\mathbf{p}}_{\mathbf{L}}^{+}\left(\mathbf{B}_{\mathrm{LK}} \underline{\mathbf{p}}_{\mathbf{K}} + \mathbf{B}_{\mathrm{LL}} \underline{\mathbf{p}}_{\mathbf{L}}\right)\right\} ,$$

where $\mathbf{p}_{\mathrm{K}}$, $\mathbf{p}_{\mathrm{L}}$ resp., are defined by row k, l resp., of the matrix $\mathbf{P}^+$

$$(22) \qquad \underline{\mathbf{P}}^{+} = \begin{bmatrix} 1 & a & a^2 \\ 1 & a^2 & a \\ a^2 & 1 & a \\ a & 1 & a^2 \\ a & a^2 & 1 \\ a^2 & a & 1 \end{bmatrix} .$$

From the eq. (20) and (21) follows that a change (increase or decrease) of the reactive power due to capacitive linkage between lines K and L can be expressed by the following relations

(23) $$\Delta Q_K = \omega U^2 \operatorname{Im}\left\{\underline{\mathbf{p}}_K^+ \mathbf{B}_{KL} \underline{\mathbf{p}}_L\right\},$$
$$\Delta Q_L = \omega U^2 \operatorname{Im}\left\{\underline{\mathbf{p}}_L^+ \mathbf{B}_{LK} \underline{\mathbf{p}}_K\right\}.$$

A change of the charging current caused by capacitive coupling between lines K and L can be evaluated from the eq. (18)

(24) $$\Delta \underline{\mathbf{I}}_K = j\omega U \mathbf{B}_{KL} \underline{\mathbf{p}}_L \qquad \Delta \underline{\mathbf{I}}_L = j\omega U \mathbf{B}_{LK} \underline{\mathbf{p}}_K$$

Tab. 2a.

Tower Soudek 110kV	q [μC/m]	M [μH/m]	E [kV/m]
× • o o • ×	0,047	0,05	0,4
× • o × • o — × o o • • ×	0,092	0,14	0,7
× o o × • •	0,130	0,22	1,4
× × o • • o	0,177	0,27	1,0
× × o o • •	0,183	0,28	1,5

Tab. 2b.

Vertically arranged conductors 400kV	q [μC/m]	M [μH/m]	E [kV/m]
× • o o • ×	0,166	0,04	1,2
× • o × • o — × o o • • ×	0,309	0,15	1,9
× o o × • •	0,432	0,25	3,7
× × o • • o	0,599	0,28	2,6
× × o o • •	0,618	0,30	3,8

Example: For three types of double-circuit overhead lines placed on towers Soudek (110 kV), Donau (440 kV) and vertically arranged conductors (440 kV) we calculate the reactive power taken-off in different arrangement of phases:

a) without respect to capacitive coupling between lines K and L
b) with respect to capacitive coupling between lines K and L

From obtained results (tab. 3) is visible that the non-symmetry of capacitive linkage between the lines causes in either increase or decrease of total reactive power in dependence on the phases configuration. The arrangement for minimum of the reactive power is in a good agreement with the configuration for minimal charges on neutral conductors.

Tab.3.

	Without linkage of K and L	Respecting the linkage of K and L			
	Q [MVAr]	Q [MVAr]		difference	
		Q_{min}	Q_{max}	[kVAr]	[%]
Soudek 110 kV	1,06	1,01	1,09	78	7,7
Donau 440 kV	12,26	12,12	12,34	217	1,8
Vertically arranged conductors 440 kV	12,90	12,27	13,27	995	8,1

5. CONCLUSION

In this paper the algorithm for calculation of the capacitive coupling in the systems with neutral conductors is presented. The geometrical non-symmetry is taken into account. The method is applied on double-circuit lines; the analysis of influence of phase conductors arrangement on relevant lines parameters (partial capacitances, charges of phase conductors and of neutral conductors, charging current and reactive power supplied by the capacitances) is carried-out. Based on the performed calculations one can posit a recommendation for optimal design of transposition of conductors in the double-circuit overhead lines. Presented results coincide with previous works of authors which were focused on evaluation of an influence of appropriate transposition of conductors in double-circuit lines on inductive coupling and on distribution of electrical and magnetic fields near earth. According to the suggested algorithm such an arrangement of the phase conductors can be found which minimize above-mentioned values.

Acknowledgement

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