\[ h_i(x, t) = \frac{M_i \phi(x, t)}{A_i(x, t) \bar{M}_i(x, t)} e^{-\frac{\sigma \sqrt{l_i} x}{2 R_i^2}}. \tag{10b} \]

The influence of the parameter \( \alpha \) on the magnetic field distribution in the cylindrical charge is shown in Fig. 2 at \( t = T/4 \), that is to say for the instantaneous value of the external magnetic field equal to its amplitude. The space-time distribution of this field is shown in Fig. 3.

Having determined the magnetic field strength \( H'(x, y) \) from (11), we can find the first Maxwell equation \( \nabla \times H'(x, y) = J(x, y) \), determine the current density \( J(x, y) = I_a \vec{l}_a(x, y) \). Then also the Poynting vector \( P(x, y) = E(x, y) \times H(x, y) = \vec{P}(x, y) \vec{I}_a \), that will let us determine the heating of the cylindrical charge in the case of the transient magnetic field.

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REFERENCES

3. M-PHASE SYSTEM WITH NEUTRAL CONDUCTORS

Let us assume a system of M phase conductors with N neutral conductors (see Fig. 3). In the similar way to eq. (1) we can describe a relation among phase-to-neutral voltages and charges of all conductors respecting the fact that \( i = 0 \) for \( M = 1, \ldots, M+N \) (neutral conductors are grounded) by matrix equation

\[
\begin{bmatrix}
    U_{i1} \\
    U_{i2} \\
    \vdots \\
    U_{iN}
\end{bmatrix}
= \begin{bmatrix}
    A_{(M+1)M} & A_{M+1} & A_{M+2} & \cdots & A_{MN} & 0 & 0 & \cdots & 0
\end{bmatrix}
\begin{bmatrix}
    q_{i1} \\
    q_{i2} \\
    \vdots \\
    q_{iN}
\end{bmatrix}
\]

From the second row of the eq. (7) we can obtain the charges of neutral conductors

\[
q_{iN} = A_{MN} q_{i1} + A_{MN} q_{i2} + \cdots + A_{MN} q_{iM+n} = 0
\]

after substituting into eq. (7) we will get a relation among voltages and charges of phase conductors.

(9) \( U_{i1} = \begin{bmatrix}
    A_{(M+1)M} & A_{M+1} & A_{M+2} & \cdots & A_{MN} & 0 & 0 & \cdots & 0 \end{bmatrix} q_{i1} \)

Now a matrix of capacitance coefficients is defined by following equation

(10) \( B = (A_{MN} - A_{MN} A_{(M+1)M})^{-1} \)

a matrix of charges of phase conductors will have the form

(11) \( q_{i1} = B U_{i1} \)

The following relation can determine the charges of neutral conductors

(12) \( q_{iN} = A_{MN} q_{i1} + A_{MN} q_{i2} + \cdots + A_{MN} q_{iM+n} \)

In accordance with relations (4) we can from eq. (10) define partial capacitances of phase conductors; by this way we get a set of capacitances corresponding with Fig. 2. The obtained capacitances \( C_i \) however respect not only the capacitances against earth but also the linkage of phase and neutral conductors.

Example: For the three-phase system (see Fig. 4a for proportions of tower, phase conductors are signed as 1, 2, 3, neutral conductor as 0) determine the effective capacitance \( C_{ij} \), charging current and reactive power supplied by capacitances. The length of the line is \( I = 10 \, \text{km} \) and the phase-to-neutral voltage \( U = 110 \, \text{kV} \).

We will perform the calculation by both ways:

a) without respect to the capacitive coupling to the neutral conductor

b) with respect to the capacitive coupling \( C_{01}, C_{02}, C_{03} \) (see Fig. 4b)

(13) \( C_i = \frac{1}{C_{i1} + C_{i2} + C_{i3}} \)

we partially respect the geometrical non-symmetry of the line.

Solution:

case a)

we will perform the calculation using eq. (2) - (6), and substituting into eq. (5).

(14) \( \frac{1}{C_{i1} + C_{i2} + C_{i3}} = \frac{1}{C_{i1} + C_{i2} + C_{i3}} \) \( i = 1, 2, 3 \)

Similarly to eq. (11) we will determine charges of phase conductors of lines K and L.

(17) \( q_{i1} = \frac{q_k}{B_{i1} B_{i1} + B_{i1} B_{i2} + B_{i1} B_{i3}} \)

Similarly to eq. (11) we will determine charges of phase conductors of lines K and L.

(18) \( q_{i1} = \frac{q_k}{B_{i1} B_{i1} + B_{i1} B_{i2} + B_{i1} B_{i3}} \)

From the eq. (10) and (11) follows that a change (increase or decrease) of the reactive power due to

\[
\begin{bmatrix}
    1 & a & a^2 \\
    1 & a^2 & a \\
    1 & a & a^2
\end{bmatrix}
\]

between lines K and L. Then we can determine charging currents from the formula

(19) \( \frac{1}{a} q_j + q_{j+1} + q_{j+1} = 0 \)

if we denote

(20) \( \begin{bmatrix}
    a & a & a \\
    a & a & a \\
    a & a & a
\end{bmatrix} \)

As a consequence of the geometrical non-symmetry there is a capacitive coupling non-symmetry and therefore for a balanced voltage system is the set of currents and charges unbalanced, which we can express by formula

With the use of \( B_{i1} B_{i1} \) instead of \( B_{i1} B_{i1} + B_{i1} B_{i2} + B_{i1} B_{i3} \) the charges induced in the neutral conductor and on the ground surface will be \( q_{i1} = q_1, q_{i2} = \frac{q_2}{1}, q_{i3} = \frac{q_3}{1} \) instead of \( q_{i1} = q_1, q_{i2} = \frac{q_2}{1}, q_{i3} = \frac{q_3}{1} \) the charges induced in the neutral conductor and on the ground surface will be \( q_{i1} = q_1, q_{i2} = \frac{q_2}{1}, q_{i3} = \frac{q_3}{1} \). The charges on the neutral conductor can be evaluated from the eq. (12), their value is proportional to non-symmetry of the capacitive coupling and can be used to determine optimal conductors transposition. Values of the charges of neutral conductors are listed for six basic arrangements according to eq. (16) in tab. 2a for the case of the tower Westend and in tab. 2b for the case of vertical conductors arrangement. The phases are denoted by general symbols \( \alpha, \beta, \gamma \) and \( \alpha' \). The total inductance of the neutral conductors and values of electric field strength in height 1.2 m above the ground are also listed in these tables – they were calculated in the paper [3]. From the comparison with these results follows that optimal transposition of conductors in double-circuit lines is also minimizing not only non-symmetry of inductive and capacitive linkage but also the electric field strength \( E \) in height 1.2 m above the ground.

The total reactive power supplied by capacitances on line K is

(21) \( \begin{bmatrix}
    1 & a & a \\
    a & a & a \\
    a & a & a
\end{bmatrix} \)

where the symbol \( \begin{bmatrix}
    U \end{bmatrix} \), \( \begin{bmatrix}
    I \end{bmatrix} \) denotes a matrix associated with the matrix \( U \) , the matrix with complex conjugated elements. The reactive power supplied on line L is similarly

(22) \( \begin{bmatrix}
    1 & a & a \\
    a & a & a \\
    a & a & a
\end{bmatrix} \)

Similarly to eq. (11) we will determine charges of phase conductors of lines K and L.

(23) \( \begin{bmatrix}
    1 & a & a \\
    a & a & a \\
    a & a & a
\end{bmatrix} \)

From the eq. (10) and (11) follows that a change (increase or decrease) of the reactive power due to
3. M-PHASE SYSTEM WITH NEUTRAL CONDUCTORS

Let us assume a system of M phase conductors with N neutral conductors (see Fig. 3). In the similar way to eq. (1) we can describe a relation among phase-to-neutral voltages and charges of all conductors respecting the fact that $i_i = 0$ for $i = M+1, \ldots, M+N$ (neutral conductors are grounded) by matrix equation

$$
\begin{bmatrix}
U_{L1} \\
U_{L2} \\
\vdots \\
U_{LM}
\end{bmatrix} =
A_{MxM}^{-1} 
\begin{bmatrix}
A_{MM} & A_{MN} & q_{L1} & \ldots & q_{LM}
\end{bmatrix}
$$

\[ M+1 \]

\[ M+N \]

\[ i \]

\[ M \]

\[ q \]

\[ Q \]

\[ \phi \]

\[ Q = 0 \]

\[ 5 \text{a} \]

\[ 5 \text{b} \]

\[ 5 \text{c} \]

\[ 5 \text{d} \]

\[ 5 \text{e} \]

\[ 5 \text{f} \]

\[ 5 \text{g} \]

\[ 5 \text{h} \]

\[ 5 \text{i} \]

\[ 5 \text{j} \]

\[ 5 \text{k} \]

\[ 5 \text{l} \]

\[ 5 \text{m} \]

\[ 5 \text{n} \]

\[ 5 \text{o} \]

\[ 5 \text{p} \]

\[ 5 \text{q} \]

\[ 5 \text{r} \]

\[ 5 \text{s} \]

\[ 5 \text{t} \]

\[ 5 \text{u} \]

\[ 5 \text{v} \]

\[ 5 \text{w} \]

\[ 5 \text{x} \]

\[ 5 \text{y} \]

\[ 5 \text{z} \]

\[ 6 \]

\[ 7 \]

\[ 8 \]

\[ 9 \]

\[ 10 \]

\[ 11 \]

\[ 12 \]

\[ 13 \]

\[ 14 \]

\[ 15 \]

\[ 16 \]

\[ 17 \]

\[ 18 \]

\[ 19 \]

\[ 20 \]

\[ 21 \]

\[ 22 \]
capacitive linkage between lines K and L can be expressed by the following relations

\[ \Delta Q = \alpha V^2 \left( \frac{1}{l_{MK}} + \frac{1}{l_{LP}} \right) \]

\[ \Delta Q = \omega V^2 \left( \frac{1}{l_{MP}} + \frac{1}{l_{KP}} \right) \]

A change of the charging current caused by capacitive coupling between lines K and L can be evaluated from the eq. (18)

\[ \Delta I_{MK} = j \omega l_{MK} B_{MK} P_{MK} \]

\[ \Delta I_{LP} = j \omega l_{LP} B_{LP} P_{LP} \]

Tab. 2a.

<table>
<thead>
<tr>
<th>Tower Sousedek</th>
<th>q</th>
<th>M</th>
<th>E</th>
</tr>
</thead>
<tbody>
<tr>
<td>110 kV</td>
<td>0.047</td>
<td>0.05</td>
<td>0.4</td>
</tr>
<tr>
<td>220 kV</td>
<td>0.092</td>
<td>0.14</td>
<td>0.7</td>
</tr>
<tr>
<td>400 kV</td>
<td>0.130</td>
<td>0.22</td>
<td>1.4</td>
</tr>
<tr>
<td>400 kV</td>
<td>0.147</td>
<td>0.27</td>
<td>1.0</td>
</tr>
<tr>
<td>400 kV</td>
<td>0.183</td>
<td>0.28</td>
<td>1.5</td>
</tr>
</tbody>
</table>

Tab. 2b.

<table>
<thead>
<tr>
<th>Vertically arranged conductors</th>
<th>q</th>
<th>M</th>
<th>E</th>
</tr>
</thead>
<tbody>
<tr>
<td>400 kV</td>
<td>0.166</td>
<td>0.04</td>
<td>1.2</td>
</tr>
<tr>
<td>400 kV</td>
<td>0.300</td>
<td>0.15</td>
<td>1.9</td>
</tr>
<tr>
<td>400 kV</td>
<td>0.432</td>
<td>0.25</td>
<td>3.7</td>
</tr>
<tr>
<td>400 kV</td>
<td>0.599</td>
<td>0.28</td>
<td>2.6</td>
</tr>
<tr>
<td>400 kV</td>
<td>0.618</td>
<td>0.30</td>
<td>3.8</td>
</tr>
</tbody>
</table>

5. CONCLUSION

In this paper the algorithm for calculation of the capacitive coupling in the systems with neutral conductors is presented. The geometrical non-symmetry is taken into account. The method is applied on double-circuit lines; the analysis of influence of phase conductors arrangement on relevant lines parameters (partial capacitances, charges of phase conductors and of neutral conductors, charging current and reactive power supplied by the capacitances) is carried out. Based on the performed calculations one can posit a recommendation for optimal design of transposition of conductors in the double-circuit overhead lines. Presented results coincide with previous works of authors which were focused on evaluation of an influence of appropriate transposition of conductors in double-circuit lines on inductive coupling and on distribution of electrical and magnetic fields near earth. According to the suggested algorithm such an arrangement of the phase conductors can be found which minimize above-mentioned values.

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REFERENCES


Comparison of systems for levitation heating of electrically conductive bodies

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Summary. Levitation heating of nonmagnetic electrically conductive bodies is realized in various systems consisting of one or more inductors. The paper deals with comparison of the resultant Lorentz lift force acting on such a body (cylinder, sphere) and velocity of its heating for different shapes of coils and parameters of the field currents (amplitude, frequency). The task is solved in quasi-coupled formulation. Theoretical considerations are supplemented with an illustrative example whose results are discussed.

1. INTRODUCTION

Levitation heating of solid electrically conductive bodies is used in a number of various modern technologies. One of them is, for example, levitation melting of metals in an inert atmosphere.

Design and optimization of the device whose fundamental part is represented by one or several field coils in an appropriate arrangement and investigation of the complete process require, however, reliable mathematical and computer models providing complete information about its characteristics and overall effect.

The most important parameters playing the fundamental role in the process are the total repulsive eddy current force acting on the body across the field and velocity of its heating. These magnitudes depend on particular disposition of the system and should be in such relations that the efficiency of the process is as high as possible.

Disposition of the field coils may differ from one case to another (their shapes can be cylindrical, conical or even more sophisticated). In various applications the system may be placed in transversal magnetic field produced by supplementary coils that provides stabilizing rotation of the processed workpiece.

The paper represents only an introductory study whose aim it to obtain a sufficient amount of information about dimensions of such a device from the viewpoint of the shape and other parameters of the field coil that would secure both the proposed position of the workpiece and its sufficiently fast heating. This task represents a coupled electromagnetic-temperature problem that is solved in a quasi-coupled formulation.

The subsequent task – computer modeling of melting of the levitated material and its behavior in applied electromagnetic field representing a coupled magnetohydrodynamic-temperature problem and associated solution of the free level of melt is now being prepared and is not included in this paper.

2. DESCRIPTION OF THE TECHNICAL PROBLEM

Consider an axis-symmetric levitating system consisting of one coil of a suitable shape (Fig. 1) in a cylindrical coordinate system r, z. The system consists of two active parts: an inductor I carrying harmonic current of amplitude I and frequency f and a well electrically conductive body 2 that is to be levitated and heated.

Fig. 1. The investigated arrangement.

At the time t = 0 the inductor I is connected to the source of harmonic current I and starts producing harmonic magnetic field. This field induces eddy currents Ë in the body 2. Interaction between the magnetic field and eddy currents induced in the body then produces the Lorentz forces that make it move up from the starting position S (Fig. 1a) in the direction of the indicated arrows. At the same time the body starts to be heated by the Joule losses. After a short transient taking time t the body reaches the end position E where the Lorentz forces acting on it are in balance with its weight (Fig. 1b).

Growing temperature of the inductor and, particularly, processed body affects the physical properties of the system, among other electrical conductivity of its parts. Its variation influences distribution of magnetic field and, consequently, the position E. On the other hand, these variations are relatively small and in most cases may be neglected.

The final step is melting of the body and eventual shaping of the melt. We will model, however, the process only in some instant t when the average temperature of the body reaches a pre-