SINGLE-PHASE POWER THEORY USING ORTHOGONAL TRANSFORMATIONS

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Summary: The paper deals with the new method of power analysis of single-phase power electronic systems. Using a new particular transform theory the ordinary single-phase system can be transformed into equivalent two-axis orthogonal one. The new original thought is based on the idea that ordinary single-phase quantity can be complemented by fictitious second phase so that both of them will create orthogonal system, as it usual in three-phase systems. Application of the above theory makes this possible to use complex methods of analysis as instantaneous reactive power method, which have not been usable for single-phase systems so far. All types of the power, active and reactive, can be determined by this way.

1. INTRODUCTION

It is well known that the analysis of multiphase systems can be more simple using the Clarke/ Park transformation in two-axis stationary (α,β) or rotary (d,q) reference frame. The above transformation can be used for electrical machines as well as for power electronic systems. The projection of time state-space vector for any quantity of symmetrical three-phase system in Gauss complex plane $(\alpha + j\beta)$ shows out six-side symmetry of vector quantity trajectory. Then, analysis of such a system can be focused on the interval equal to 1/6 of the time period only [1]-[3]. It is clear that when using similar transform of single-phase quantity into equivalent two-axes orthogonal system it will be possible to use all advantages as in three-phase transformed system with respect of 4-side symmetry instead of 6-side of previous[4].

2. USING ORTHOGONAL TRANSFORMATION FOR SINGLE-PHASE SYSTEM

As mentioned the basis for this approach can be symbolic vector expression and substitution of harmonic function,

$$\cos(\omega t) \rightarrow \exp(j\omega t) = \cos(\omega t) + j.\sin(\omega t), \quad (1)$$

thus for resistant-inductive load current in steady-state

$$i^*(t) = U.\exp(j\omega t)/|Z|.\exp(j\varphi) = I.\exp[j.(\omega t-\varphi)]$$
$$= I.[\cos(\omega t-\varphi)+j.\sin(\omega t-\varphi)], \text{ where} \quad (2)$$

$Z = R+j\omega L$, $I = U/|R+ j\omega L|$, and

$\varphi = \text{arctg}(\omega L/R)$.

The resulted current is simply the real part of $i^*(t)$, i.e.:

$$i(t) = I.\cos(\omega t-\varphi). \quad (3)$$

Assuming a single-phase system defined as in Fig. 1a

$$u(t) = U.\cos(\omega t); i(t) = I.\cos(\omega t-\varphi). \quad (4)$$

After complementing by fictitious imaginary phase defined as in [4] by above approach as Fig. 1b

$$u_i(t) = U_i.\sin(\omega t); i_i(t) = I_i.\sin(\omega t-\varphi), \quad (5)$$

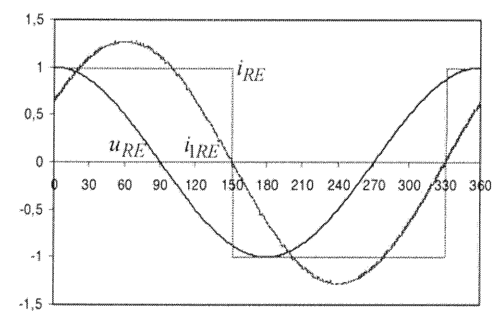

Fig. 1a Example of time-waveforms of voltage and current of the real phase

we obtain orthogonal co-ordinate system whereas

$$u_\alpha = u(t) \text{ and } u_\beta = u_i(t). \quad (6)$$

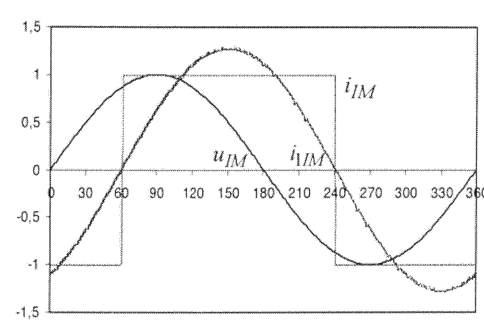

Fig. 1b Time-waveforms of voltage and current of the fictitious imaginary phase

Note that both phases, real- and imaginary ones, are completely separated, however they are synchronised by signal SYNC, see Fig. 2. Such arrangement implies that zero component of any quantity will be *a priori* zero.

Generally, the fictitious phase can be created by shifting of ordinary single-phase quantity to the right with phase shift equal to $-\pi/2$. It follows out from the 4-side symmetry of vector quantity trajectory in Gauss plane [4].

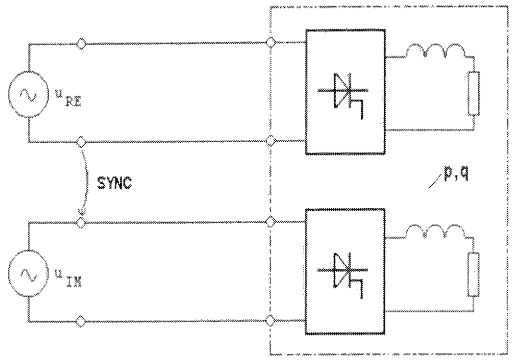

Fig. 2 Arrangement of real- and fictitious imaginary phases of single-phase system

The following equations (7)-(9) must be valid for quantity with 4-side symmetry

$x^*(t) = x^*(t\text{-T}/4).\exp(\mathrm{j}\pi/2)$, and also (7)

$x(t) = -x(t\text{-T}/2)$ and $x_i(t) = x(t\text{-T}/4)$. (8)

Finally, the general transform equation can be introduced for single-phase system

$x^*(t) = \mathrm{K}.[x(t) + \exp(\mathrm{j}\pi/2).x_i(t)]$, (9)

where K is multiplicative constant (equal to 1 for single-phase system) and

$x_\alpha = x(t)$ and $x_\beta = x_i(t)$. (10a,b)

Let's notice that all integral quantities (e.g. average- and/or rms values) are then possible to compute over one fourth of time period.

2.1. Fourier Analysis in Orthogonal Coordinate System

Fourier analysis of investigated quantity is also possible to do in 1/4 of time period.

The complex Fourier coefficients for basic harmonic component are defined now as

$$C_1 = \frac{4}{T}\int_0^{\frac{T}{4}} x(t).e^{-j\omega t}dt\,, \qquad (11)$$

that means, within one fourth of the time period. The magnitude and phase shift of the fundamental harmonic component of any quantity $x(t)$ is then:

$$C_1 = \sqrt{(C_{1\alpha}^2 + C_{1\beta}^2)}\,,\ \varphi_1 = \arctan\frac{C_{1\beta}}{C_{1\alpha}}. \qquad (12,13)$$

Let's assume a set of numerical data for $u(t)$ and $u_i(t)$ from [4]. The result can be gained by application of numerical solution of the integral expression (11) by substitution

$_0\int^{T/4} x^*(t).\exp(-\mathrm{j}\omega t).dt \approx$

$1/\mathrm{N}._{k=1}\Sigma^{\mathrm{N}-1}[f(\mathrm{k}.\Delta T) + (f(0)+f(\mathrm{N}))/2]$, (14)

where N is number of samples, k is order of the sample.

3. MODIFIED SINGLE-PHASE POWER THEORY

3.1. Instantaneous Active-, Reactive- and Distortion Power Determination for Both Real- and Imaginary Phases under Linear Load (with Inductive/Capacitive Power Factor)

Assume now, for the simplicity, harmonic waveforms of phase-voltage and phase-current

$u(t) = U.\cos(\omega t); i(t) = I.\cos(\omega t\text{-}\varphi)$. (15a,b)

Utilisation of instantaneous reactive power method is used in [5] for three-phase systems, and above theory allows its use for single-phase systems as well taking in account (4) - (6)

$p_{\alpha\beta} = u_\alpha.i_\alpha + u_\beta.i_\beta,$

$q_{\alpha\beta} = u_\alpha.\ i_\beta - u_\beta.\ i_\alpha.$ (16a,b)

Note that instantaneous power $p_{\alpha\beta}$ is not purely active one because of it comprises both DC-average and AC alternating components generally

$p_{\alpha\beta} = P_{\alpha\beta\mathrm{AV}} + p_{\alpha\beta\mathrm{AC}}.$ (17)

In fact, $p_{\alpha\beta}$ is apparent power of both real- and fictitious phases.

Similarly as in (17) we get for instantaneous power $q_{\alpha\beta}$

$q_{\alpha\beta} = Q_{\alpha\beta\mathrm{AV}} + q_{\alpha\beta\mathrm{AC}},$ (18)

where $q_{\alpha\beta}$ are the instantaneous reactive- and distortion components of powers of both phases in orthogonal co-ordinates.

Power components for non-linear load (diode rectifier) are shown in Fig. 3.

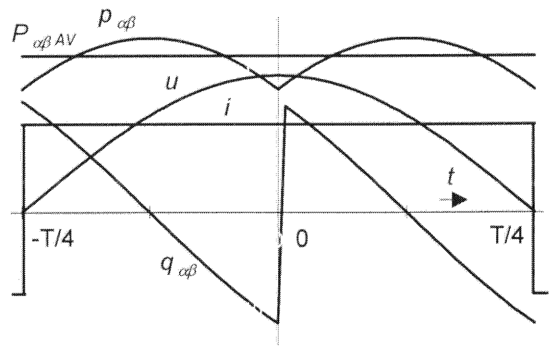

Fig. 3 Time-dependence of instantaneous $p_{\alpha\beta}$, $q_{\alpha\beta}$ components of the power for non-linear load

3.2. Instantaneous & Average Values of Active, Reactive and Distortion Power Determination of Real Phase under Linear Load (with Inductive Power Factor)

From the Fig. 5 flows that active- and reactive power average values of real phase will be simply one half of $p_{\alpha\beta}$ and $q_{\alpha\beta}$, respectively. Instantaneous values of active- and reactive power components of real phase then will be

$p_1 = p_{\alpha\beta}/2.[1+\cos(2\omega t)] =$

$= p_{\alpha\beta}[(\cos^2(\omega t)],$ (19a)

and $q_1 = q_{\alpha\beta}/2.[1\text{-}\cos(2\omega t)] =$

$= q_{\alpha\beta}[(\sin^2(\omega t)].$ (19b)

Note that instantaneous active power p_1 is not equal instantaneous product of phase voltage and phase current

$s = u_\alpha.i_\alpha\,,$ (20)

which is instantaneous apparent power, in spite of its average value is equal to average value of active power. Instantaneous reactive power q_{RE} is to be understood as negative one due to presenting below time axis for clarity.

Instantaneous distortion power $p_{\alpha\beta AC}$ and $q_{\alpha\beta AC}$ in both axes will be equal zero.

Average values of active- , reactive- and distortion power are then constant due to constant waveforms of (17), (18)

$$P_{AV} = P_{\alpha\beta AV}/2 \quad \text{and} \quad Q_{AV} = Q_{\alpha\beta AV}/2. \qquad (21a,b)$$

Distortion component of power will be zero due to absence of high harmonic components:

$$D_{AV} = 0. \qquad (22)$$

The apparent power comprises all power components by the relation

$$S = \sqrt{(P^2_{AV}+Q^2_{AV}+D^2_{AV})}. \qquad (23)$$

Phase displacement factor of fundamental harmonic component can be expressed as

$$\varphi_1 = \arctan (Q_{AV} / P_{AV}) =$$
$$= \arctan [(u_{\alpha}. i_{\beta} - u_{\beta}. i_{\alpha})/(u_{\alpha}.i_{\alpha} + u_{\beta}.i_{\beta})],(24)$$

and it can be determined without any "zero-crossing" measurement.

Power relations for all power components of real phase are shown in Fig. 4

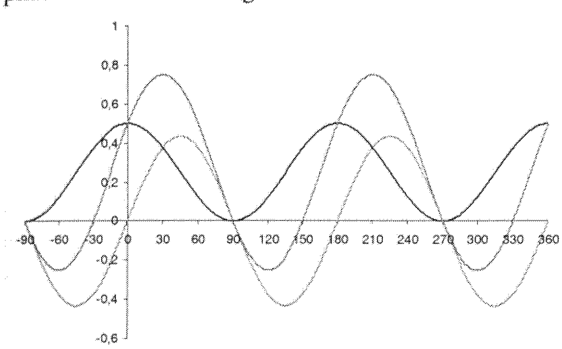

Fig. 4 Power component waveforms of real phase in p-axis

It's important that p_1, q_1 and φ_1 are – under condition of steady-state and linear load - determined instantaneously, what is the essential contribution of introduced method in relation to so far known ones.

The total apparent power s (equal $u_{\alpha}.i_{\alpha}$) can be generally projected and decomposed into p-, q- and r-axes. But, under p-projection the average value of all reactive power will be zero (Fig. 7), and, vice-versa under q-projection, the average value of active power will be zero (Fig. 8). Power in r-axis (distortion component) will be zero in case of linear load. In contrary to p- and q-axes it comprises reactive high harmonics power, only. Average values of both component of fundamental power $p1$ and $q1$ will be therefore also zero.

Generally, the power in p-axis will be consist of active component p_1 and distortion component of power $p_{\sim}$, what is analogically to (17), (18)

$$p = p_1 + p_{\sim} , \qquad (25)$$

similarly as power in q-axis

$$q = q_1 + q_{\sim} . \qquad (26)$$

The q-power components are shown in Fig. 5.

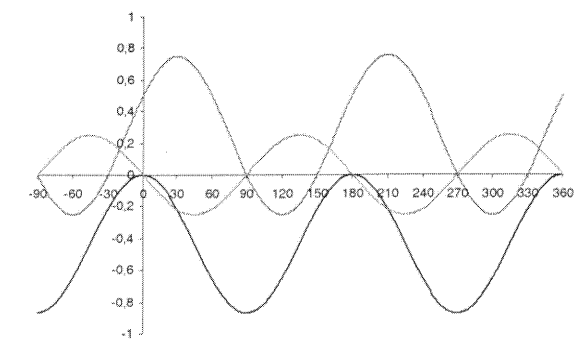

Fig. 5 Power component waveforms of real phase in q-axis

3.3. Instantaneous & Average Values of Active, Reactive and Distortion Power Determination of Real Phase under Non-Linear Load with Inductive/Capacitive Power Factor

In case of non-linear loads the values of instantaneous active and reactive powers are not constant, due to existence of distortion power caused by higher harmonic components.

Let's consider non-linear waveforms of current as in Fig. 6.

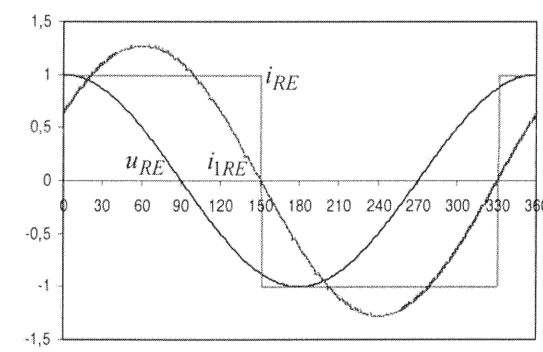

Fig. 6 Example of time-waveforms of voltage and current of the real phase

Power circumstances for power components of both phases are shown in Fig. 7

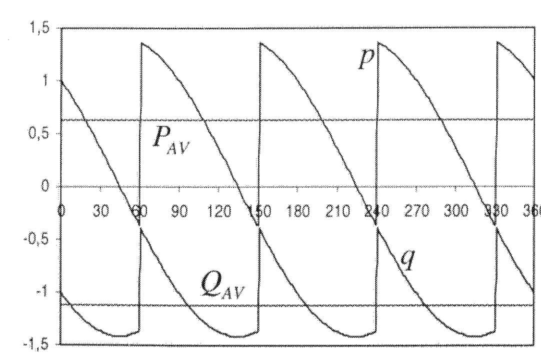

Fig. 7 Active- and reactive powers of both phases at non-linear load

Power component in p-axis of real phase can be shown now as in Fig. 8.

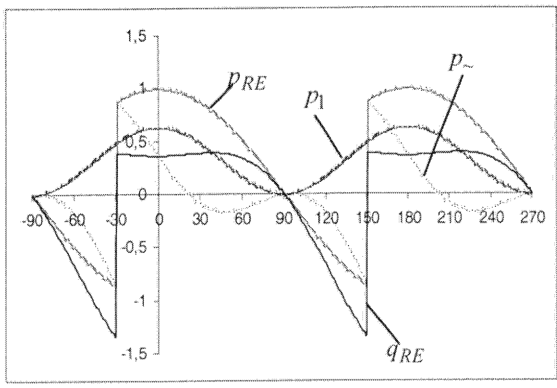

Fig. 8 Power component waveforms of real phase in p-axis

Using time-sub-optimal analysis in transformed orthogonal co-ordinates for 4-side symmetry an average value of active power P_{AV} of an original (real) phase

$$P_{AV} = P_{\alpha\beta AV}/2 = 2/T.{}_0\int^{T/4}[u_\alpha.i_\alpha + u_\beta.i_\beta\,].dt, \quad (27)$$

and reactive power Q_{AV} of original phase

$$Q_{AV} = Q_{\alpha\beta AV}/2 = 2/T.{}_0\int^{T/4}[u_\alpha.\, i_\beta - u_\beta.\, i_\alpha].dt. \quad (28)$$

Phase displacement is also possible to determine by average values of active- and reactive powers

$$\varphi_1 = \arctan(Q_{AV} / P_{AV}). \quad (29)$$

Now, the phase displacement is integral quantity and can be determined over one fourth of the time period similarly as average power values.

Power component in *p*-, *q*- and *r*-axes of real phase can be shown now as in Fig. 8, Fig. 9 and Fig. 10.

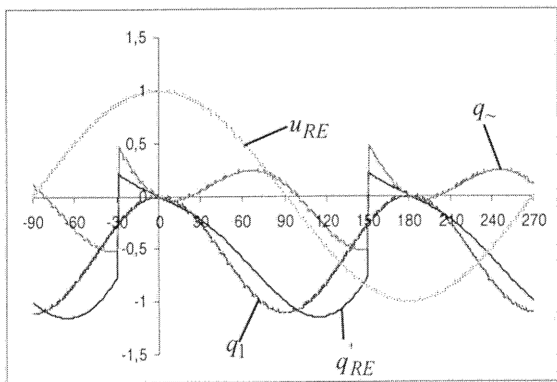

Fig. 9 Power component waveforms of real phase in q-axis

In case of ideal rectagonal waveform of load current (idealized diode rectifier) the actual value of Q_{AV} will be zero due to zero phase displacement φ_1.

For $\varphi_1 = 0$ (and $Q_{AV} = 0$), the distortion component of power can be calculate as

$$D_{AV} = \sqrt{(S^2 - P^2_{AV})} = \sqrt{[(2/\pi)^2 - (1/2)^2]} = 0.394 \text{ p.u.} \quad (30)$$

Average values of active and reactive powers of imaginary fictitious phase P_{iAV}, Q_{iAV} can be determined by similar way.

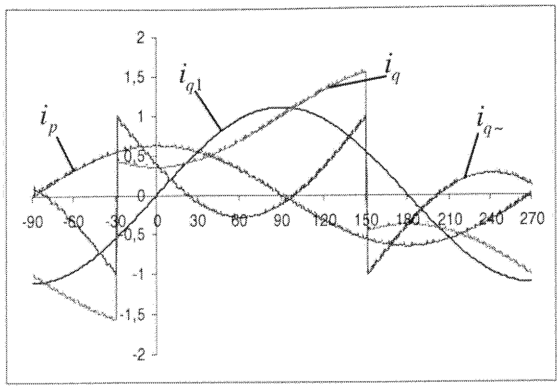

Fig. 10 Active- and reactive components of the real phase current

4. DISCUSSION AND CONCLUSION

In active filters the reference values of compensating currents are based on the knowledge of above derived formulae for active and reactive powers, where the average value of active power must be eliminated. In three-phase active filters the AC components of powers are gained usually by low-pass filtering, sometimes by calculation through some time interval (one period). Thanks to introduced theory is possible to compute the average values of active and reactive powers for 1/4 of time period. These average values can be calculated continuously for each time instant *t*, using data stored for previous 1/4 of period.

Acknowledgement

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