

## WAVELET DENOISING OF NMR SIGNAL USING QMF FILTER BANK DESIGNED BY REMEZ ALGORITHM

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**Summary** The wavelet transform is an up-to-date method for digital signal processing used in many branches of technology. One of its applications is the suppression of noise in useful signal. The paper deals with suppressing noise in a signal scanned on the NMR tomograph. The method of sub-band thresholding using the wavelet transform is discussed. This method is used in double denoising filtering of an FID signal and an instantaneous frequency signal. Using a filter bank with uniform ripple, designed by the Remez algorithm, is of advantage.

### 1. INTRODUCTION

The physical phenomenon in which the magnetic field of the nuclei of some atoms of the substance under examination reacts mutually with a rotating magnetic field is called Nuclear Magnetic Resonance (NMR). NMR spectroscopy and tomography require the basic magnetic field to be generated with a high homogeneity [1], [2]. When generating gradient magnetic fields, undesirable phenomena appear that have to be corrected by compensation methods [3]. One of the most frequently used compensation methods consists in pre-emphasis filtering [4], [5]. To calculate the coefficients of pre-emphasis filters, it is necessary to measure the gradient field decay accurately and for as long as possible. Since the magnitude of the NMR signal being scanned gradually falls to zero while the magnitude of undesirable noise remains constant, the noise substantially degrades the measuring results. Suppressing noise in the NMR signal being measured thus becomes the basic operation in determining the pre-emphasis coefficients [5], [6].

After transferring the NMR signal being measured into the basic band and after filtering with an anti-aliasing filter, the A/D conversion is performed. This will yield a free induction decay (FID) signal. The magnitude of magnetic field is directly proportional to the instantaneous frequency (IF) of the FID signal. To calculate the IF of an FID signal, the differentiation of IF using Newton's differentiation formula is applied [7]. The FID signal and the IF signal contain noise, which needs to be suppressed. The block diagram of double wavelet filtering realizing the denoising filtering of the FID and the IF signal is given in Fig. 1. The input FID signal is subjected to denoising filtering in block WF1. Subsequently, the IF in block IFC is calculated. In the end, the noise of IF signal is suppressed in block WF2.

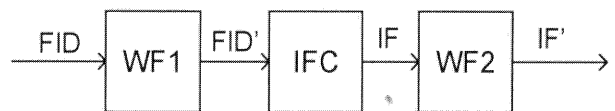


Fig. 1 Block diagram of denoising filtering.

The inner arrangement of blocks WF1 and WF2 realizing denoising filtering can be seen in Fig. 2. Using an Analysis Filter Bank (AFB) the input signal is subdivided into a number of sub-band signals. Partial sub-band signals are then thresholded such that the noise is optimally suppressed and the useful signal is not affected. Individual threshold magnitudes  $p_i$  are calculated in the Threshold Estimation (TE) block. The sub-band filters are then again synthesized in the Synthesis Filter Bank (SFB) and the output signal is formed.

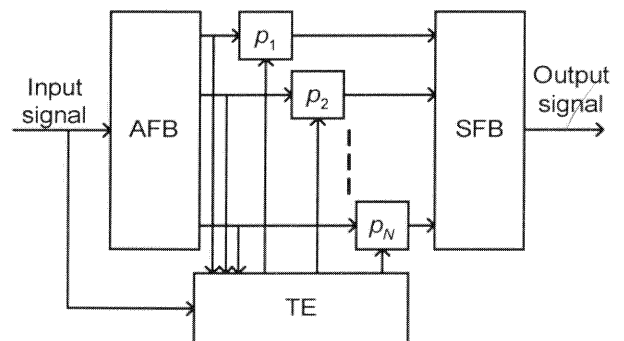


Fig. 2. Block diagram of blocks WF1 and WF2.

The choice of the parameters of individual parts of blocks WF1 and WF2 from Fig. 2 depends in the first place on the input signal properties. The filter bank base is chosen in dependence on the distribution of

input signal spectral density. The length of pulse characteristics and the type of filter bank are chosen with a view to the required attenuation in the stop band, computation complexity, and the potential occurrence of transition phenomena. To suppress noise, filter banks without downsampling are usually used. Using a filter bank with downsampling will reduce the number of operations and the application demands on memory but at the expense of worse results. The threshold magnitudes calculated in block TE are usually calculated from the standard deviations of noise  $\delta$ . Perhaps the best known relation derived for white additive noise has been that with Gaussian distribution for global threshold  $p = \delta \sqrt{2 \ln(L)}$ , where  $L$  is the input signal length. This value is usually very high, it is necessary to choose the threshold magnitude to be  $p = \delta K$ , where  $K$  is the empirically obtained constant. There are many types of thresholding, in this paper only soft or hard thresholding is chosen.

The principle of denoising filtering is identical in both blocks WF1 and WF2. They only differ in using different types of filter bank, thresholding and, above all, in the method of calculating the thresholds. This is primarily due to the different properties of the noise contained in the FID and IF signals.

## 2. DENOISING OF FID SIGNAL

Fig. 3 shows an FID signal. It is a complex signal with falling amplitude. The noise contained in the FID signal is stationary and non-white, consequently the SNR gradually decreases. With regard to the roughly exponential distribution of spectral density of FID signal, a filter bank with octave band division is used, i.e. the wavelet base. Since noise in FID signals is stationary, i.e. its properties do not change with time, the threshold magnitudes  $p_i$  are constants. The threshold magnitudes  $p_i$  are calculated from the magnitude of standard deviation of noise  $\delta$  in individual sub-band signals. The standard noise deviation  $\delta$  is calculated at the end of the FID signal being measured, when the amplitude of useful signal is zero. Soft thresholding is applied, the main advantage of which is that there are no step changes in the thresholded sub-band signal. To prevent a distortion of FID signal, which will still be processed, the  $K$ -multiple of standard noise deviation  $\delta$  is chosen comparatively small, in our case  $K = 1.5$ . The type of the filter bank used is not very important, it is only necessary to choose a sufficient order of the filters in order to prevent the signal spilling over among individual bands. The magnitude of the signal spilling over between individual sub-band filters should be less than the magnitude of noise. Using filter banks of the Daubechies type of the order of about 40 has been found advantageous [8], [9].

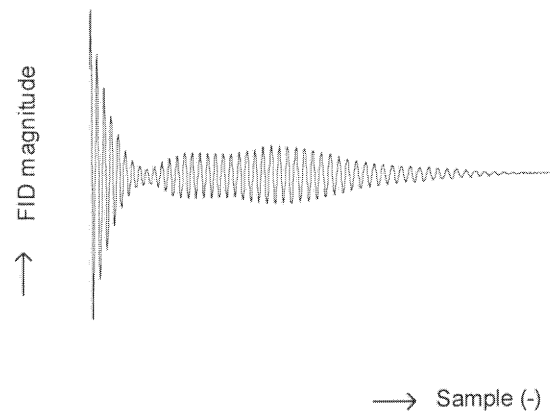


Fig. 3 Free induction decay (FID) signal.

After filtering the FID signal in block WF1 the changes are noticeable only after closer examination (Fig. 4) while the changes in the instantaneous frequency signal (Fig. 5) are clear to see. This is due to the calculation of IF signal via differentiating the instantaneous phase, during which the noise of FIG signal increases. In the first half, where the SNR of FID signal is high, there were no pronounced changes. By contrast, in the second half, where the SNR of FID signal is low, there was a considerable suppression of noise.

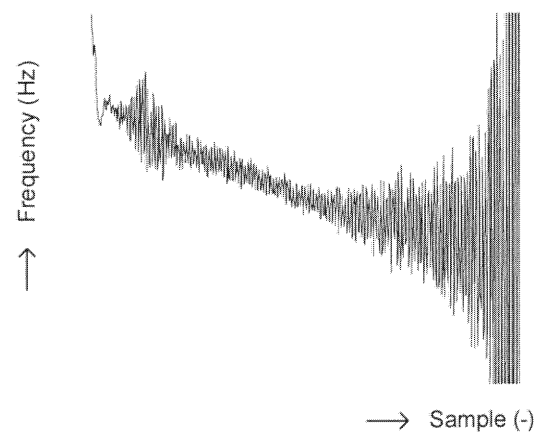


Fig. 4 Instantaneous frequency of FID signal (without filtering).

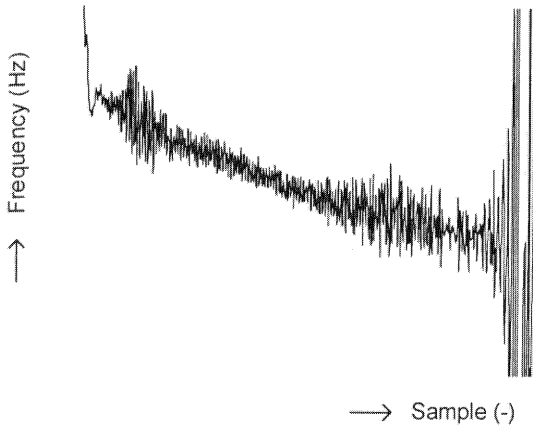


Fig. 5 Instantaneous frequency of FID signal after the first filtering by block WF1.

### 3. DENOISING OF IF SIGNAL

In the second part, when we try to suppress the noise of IF signal in block WF2, the filter bank with octave spectrum division, i.e. the wavelet base, is used in view of the spectral density distribution. The noise contained in an IF signal is strongly non-stationary, it increases gradually. Due to this, the threshold magnitude  $p_i$  must change in view of the instantaneous magnitude of the standard deviation of noise  $\delta$  in individual bands. The calculation of thresholds  $p_i$  is a very complicated operation and it usually cannot be done without human intervention, which basically consists in defining the difference between useful and harmful noise. Using a combination of thresholding and cancelling entire sections of sub-band signals also proved to be of advantage. Since thresholds  $p_i$  change with time, there are step changes in the sub-band signals. Any change, in particular a step change, is accompanied by a transient event. The length and magnitude of transient event are directly related to the filter bank used. Filter banks of low orders (Haar, Daubechies of the order of 2 to 5) are therefore applied. In this way, transient events are eliminated but, of course, at the expense of signals spilling over between individual bands, since attenuation in the stop-band of these filters is small. For this reason, we must choose a compromise between suppressing the useful spill-over signal with noise and retaining both of them.

The problem of low attenuation in the stop-band in filter banks of the Haar, Daubechies and similar type can be solved by using other filter banks with higher attenuation in the stop-band. Using filter banks designed by means of the Remez algorithm can be seen as optimum [8], [9], [10]. One advantage is evidently the highest possible suppression for a given tolerance field and filter order, another is the possibility of choosing directly the tolerance field of the filter bank. Fig. 6 gives the magnitude characteristic of a filter bank designed with the aid of the Remez algorithm while

Fig. 7 gives the sub-band signals obtained using the Remez filter bank and the Daubechies filter bank. It can be seen that the sub-band filter obtained using the Remez filter bank contains much less noise. Thanks to the greater separation of individual sub-band signals, the IF signal obtained has more details without noise.

In the WF2 block the hard thresholding is used, the main advantage of which is that it does not change the magnitude of thresholded signal. Its disadvantage, i.e. step changes in the thresholded signal, is suppressed by using a filter bank of low order.

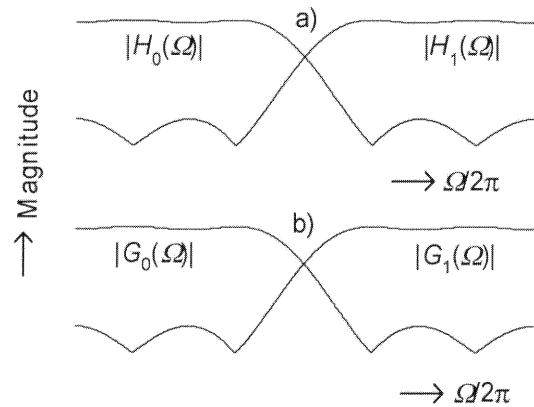


Fig. 6. Magnitude characteristics of a filter bank designed using the Remez algorithm  
a) analysis b) synthesis

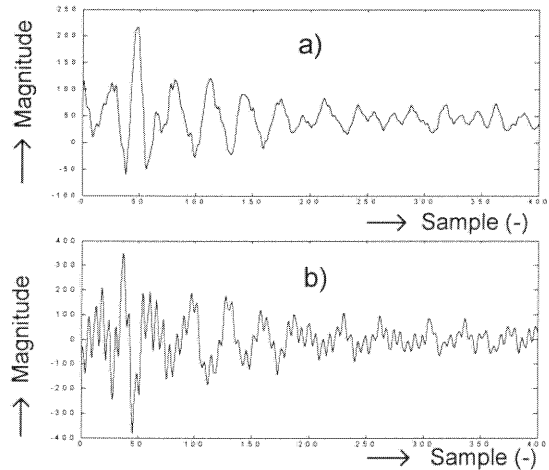


Fig. 7. Fifth subband signal obtained by a) Remez filter bank  $N = 6$  b) Daubechies filter bank  $N = 6$ .

### 4. CONCLUSION

Fig. 8 gives the IF signal after double wavelet filtering. It can be seen that considerable noise suppression has taken place. A signal of a useful length of ca. 900 samples has been obtained, which is sufficient for the calculation of the coefficients of pre-emphasis filters for the compensation of the decay of gradient magnetic field.

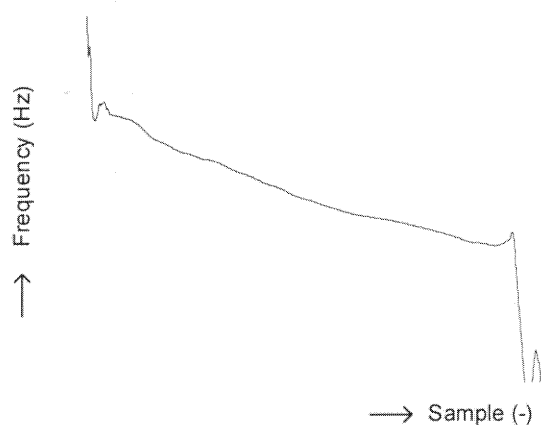


Fig. 8. Instantaneous frequency of FID signal after double wavelet filtering.

### Acknowledgement

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### REFERENCES

- [1] VLAARDINGERBROEK, Magnetic Resonance Imaging. Springer-Verlag, 2000.
- [2] P. MANSFIELD, B. CHAPMAN, "Active Magnetic Screening of Gradient Coils in NMR Imaging", *Journal of Magnetic Resonance* 66, pp. 573-576, 1986.
- [3] K. BARTUŠEK and V. PUCZOK, "The MULTIFID Method for Measurement of Magnetic Field Gradients", *Meas .Sci. Technol.* vol. 4, pp.357, 1993.
- [4] K. BARTUŠEK, B. JÍLEK, "Measurement of the Gradient Magnetic Field for NMR Tomography", in Proceedings of the 1st Nottingham Symposium on Magnetic Resonance in Medicine, Nottingham, 1994.
- [5] K. BARTUSEK, E. GESCHIEDTOVA, "Instantaneous Frequency of Spin Echo Method for Gradient Magnetic Field Measurement in MR Systems", *Journal of Electrical Engineering*, vol. 53, pp.49-52, 2002.
- [6] K. BARTUSEK, E. GESCHIEDTOVA, "Adaptive Digital Filter for Gradient Magnetic Field Measurement in MR Tomography", in Proceedings of the IEEE International Conference APCCAS' 2002, pp.79-82, 2002.
- [7] K. REKTORYS, *Přehled Užití Matematiky II.*, Prometheus, Praha 2000. (In Czech)
- [8] R. VÍCH, Z. SMÉKAL, "Digital Filters", Academia, Praha 2000. (In Czech)
- [9] N. J. FLIEGE, "Multirate Digital Signal Processing", John Wiley & Sons, Chichester 1996.
- [10] S. K. MITRA, "Digital Signal Processing", McGraw-Hill, 1998.