Behavior of a Pentacle Connected Five-Phase IM Supplied by a Rectangular Voltage

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Abstract. The presented article deals with the steady-state analysis of five-phase Induction Motor (IM), which is supplied by rectangular voltage from a five-leg Voltage Source Inverter (VSI). The new proposed approach is derived from complex Fourier-series using to express inverter terminal voltages. Based on mathematically described inverter output voltages, a formula for VSI output space phasor was derived. Assuming sinusoidal motor winding distribution and on the basis of motor parameters, stator and rotor current space phasors for defined motor load were determined. Finally, the electromagnetic torque ripple waveforms for different IM operation states were investigated.

Keywords

Complex Fourier series, five-phase inverter, induction machine, torque ripple, rectangular voltage.

1. Introduction

Variable speed electric drives, in general, preferentially utilize three-phase induction machines. Providing that the variable speed Alternating Current (AC) drives require a power electronic converter, the number of machine phases is practically unlimited. This fact led to an increasing interest for multiphase AC drives applications. Most often, they are five-phase induction machines that, by their nature, offer some advantages over their three-phase counterparts.

E. E. Ward and Harrer [1], for the first time in 1969, have presented the preliminary investigation on inverter fed five-phase IM and suggested that the amplitude of torque pulsation could be reduced by increasing the number of phases [2]. Major advantages of using a five-phase machine instead of the three-phase one consist of its higher torque density, greater efficiency and fault tolerance [3], [4] and [5]. Other advantages include reduced electromagnetic torque pulsation and reduction in the required rating per inverter leg. Noise characteristics of the five-phase drives are better when compared with the three-phase ones. For this reason, it is expected to use them in residential areas and hospitals where noise presents an undesirable element.

In the majority of cases, the supply for a multiphase variable-speed AC drives is provided by a Voltage Source Inverter (VSI) [6]. Output voltage control has a significant effect on the electromagnetic torque ripple of the motor. Control of VSI is provided in prevail by Pulse Width Modulation (PWM) techniques. Space Vector PWM (SVPWM) has become the most popular because of its easy digital implementation. However, some industrial applications of the five-phase drives do not need to modulate the output voltage. Because the rectangular voltage supply causes the ripple in motor’s electromagnetic torque, the inverter control is easier and reduces requirements on semiconductor switching units [7], [8], [9] and [10].

2. Modeling of a Five-Phase VSI

The power circuit topology of a five-phase source inverter is shown in Fig. 1. VSI consists of a parallel connection of five transistor legs denoted (a, b, ..., e). It is supplied by a constant voltage source provided by an isolated Direct Current (DC)-source and a capacitive DC-link. Each leg is composed of two Insulated-
Gate Bipolar Transistor (IGBT) transistors with anti-parallel connected free-wheeling diodes used to ensure a negative current path through the switches. Inverters output terminals are numbered (1, 2, ..., 5) [11].

To build a mathematical model of VSI, the complex Fourier series were used. They are extremely useful as a way to break up an arbitrary periodic function into a set of simple terms that can be solved individually and then recombined to obtain the solution to the original problem.

In the next, we assume idealized semiconductor devices which satisfy the following properties:

- Power switches can handle unlimited current, and they are able to block unlimited voltage.
- Voltage drop across the switch and leakage current are zero.
- The switches are turned on and off with no rise and fall times.
- Inverter input capacity is sufficiently high so we can suppose the converter input DC voltage constant for any output currents.

These assumptions simplify analysis of the power circuit and help to build against mathematical model.

The transistors in each leg are switched so that they form voltage impulses over the half period of the desired output frequency. The voltage impulse of the first transistors leg measured against the negative pole of the DC link can be expressed as [12], [13] and [14]:

$$u_a = u_{dc} + u_{01} = \frac{U_e}{2} + 2U_e \Re \left( \sum_{k=1}^{\infty} c_k e^{jk\omega t} \right), \quad (1)$$

where $u_{dc}$ is DC and $u_{01}$ is AC voltage component, $U_e$ is DC link voltage and $c_k$ is Fourier coefficient defined as:

$$c_k = \frac{1}{j2k\pi} \left( 1 - e^{-jk\pi} \right) \quad \text{valid for } k \neq 0. \quad (2)$$

Figure 2 shows the first leg output voltage waveform calculated on the base of Eq. (1). This was drawn for DC supply $U_e = 350$ V and output frequency $f = 50$ Hz.
express them as:

\begin{align}
  u_b &= a u_a, \quad u_c = a^2 u_a, \\
  u_d &= a^3 u_a, \quad u_e = a^4 u_a. \\
\end{align}

(3)

Motor phase voltages are given by the difference of voltages between two legs. Since

\begin{align}
  u_{dc} &= a u_{dc}, \quad u_{dc} = a^2 u_{dc} = u_{dc} \text{ (DC voltage shift)},
\end{align}

the phase voltages can be expressed as follows:

\begin{align}
  u_1 &= u_a - u_c = (1 - a^2) u_{01}, \\
  u_2 &= u_c - u_e = (a^2 - a^3) u_{01}, \\
  u_3 &= u_e - u_b = (a^3 - a) u_{01}, \\
  u_4 &= u_b - u_d = (a - a^3) u_{01}, \\
  u_5 &= u_d - u_a = (a^3 - 1) u_{01},
\end{align}

(4)

where \( u_{01} = 2 U_e \Re \left( \sum_{k=1}^{\infty} c_k e^{jk\omega t} \right) \) is the AC component of the voltage \( u_a \).

Figure 3 shows a phasor diagram of the pentacle connected five-phase voltage system. The phase voltages are mutually shifted by \( 2 \pi / 5 \). These form a second-order voltage system.

On the Eq. (4), the space phasor transformation is thus defined as published in [15, 16] and [17]:

\begin{align}
  \mathbf{u} = \frac{2}{5} \left[ \Re(u_1) + \Re(u_2)a_1 + \Re(u_3)a_1^2 + \\
  + \Re(u_4)a_1^3 + \Re(u_5)a_1^4 \right].
\end{align}

(5)

where \( a_1 = e^{j2\pi/5} \) is the space shifting factor for the second-order system.

Figure 3 depicts the motor phase voltage waveform calculated on the base of Eq. (1) and Eq. (4).

To simplify the calculation of AC motor quantities, it is advantageous to employ space phasors. This transformation is very often used for the analysis of multiphase electric systems. The term "space" originally stands for the two-dimensional complex plane, in which the multi-phase quantities are transformed.

The transformation of space phasor is directly derived from the sum of voltage phase phasors. Based on the Eq. (1), the space phasor transformation is thus defined as published in [15, 16] and [17]:

\begin{align}
  \mathbf{u} = \frac{2}{5} \left[ \Re(u_1) + \Re(u_2)a_1 + \Re(u_3)a_1^2 + \\
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Coefficient \( \frac{2}{5} \) keeps the magnitude of the phasors during the transformation constant.

In the Fig. 3, there is shown the VSI output voltage space phasor trajectory. The green lines represent the path of voltage space phasor jump.
The real and imaginary parts of the voltage space phasor can be separated and rewritten as:

\[ u = \text{Re}(u) + i\text{Im}(u) = u_\alpha + u_\beta. \]  

(6)

By using space phasor, the multi-phase components are transformed into a two-dimensional coordinate system. They are shown in Fig. 6.

Fig. 6: Two-dimensional voltage system components.

For further calculations, it is necessary to perform harmonic analysis of the two-phase voltage system. The following equations apply to the amplitudes of harmonics of two-phase voltage system components \( A_{\alpha k} \) and \( A_{\beta k} \), where \( k = 1, 2, 3, \ldots \):

\[ A_{\alpha k} = \frac{2}{3} \abs{\text{Re}(u_1) + \text{Re}(u_2) a_1 + \text{Re}(u_3) a_1^2 + \text{Re}(u_4) a_1^3 + \text{Re}(u_5) a_1^4}, \]

\[ A_{\beta k} = \frac{2}{3} \abs{\text{Im}(u_1) + \text{Im}(u_2) a_1 + \text{Im}(u_3) a_1^2 + \text{Im}(u_4) a_1^3 + \text{Im}(u_5) a_1^4}. \]

(7)

For each phase of harmonics, the phase shift is calculated as follows:

\[ \phi_{\alpha k} = \arg\left[\text{Re}(u_1) + \text{Re}(u_2) a_1 + \text{Re}(u_3) a_1^2 + \text{Re}(u_4) a_1^3 + \text{Re}(u_5) a_1^4\right], \]

\[ \phi_{\beta k} = \arg\left[\text{Im}(u_1) + \text{Im}(u_2) a_1 + \text{Im}(u_3) a_1^2 + \text{Im}(u_4) a_1^3 + \text{Im}(u_5) a_1^4\right]. \]

(8)

Figure 7 depicts calculated harmonic components of the two-phase voltages. Voltage waveforms are composed of 1, 9, 11, 19, 21, 29, 31, \ldots harmonics. Waves of the 1, 11, 21, 31, \ldots form positive and 9, 19, 29, 39, \ldots negative harmonics voltage sequences. There are no zero voltage sequences.

For the harmonic components which form positive voltage sequences, the following relationship is applied:

\[ k_p = 1 + 10 (n - 1), \quad n = 1, 2, 3, \ldots \]

(9)

For negative voltage sequences:

\[ k_n = 10 n - 1, \quad n = 1, 2, 3, \ldots \]

(10)

3. Motor Currents Calculation

For the stator and rotor current space phasors calculation, a classical equivalent circuit of IM shown in Fig. 8 was used.
rotor space phasor:

\[ i_1 = \sum_{k=1}^{\infty} i_{1k}, \quad i_2 = \sum_{k=1}^{\infty} i_{2k}. \]  

(13)

Calculated stator and rotor current space phasors trajectories are shown in the Fig. 8. The computation was made for nominal motor load and speed (2850 rev-min\(^{-1}\)). The motor parameters are listed in App. A.

4. Electromagnetic Torque Calculation

Electromagnetic torque of IM can be determined from the stator or rotor current values according to the formula [18] and [19]:

\[ M_{em} = \frac{5}{2} p (\psi_1 \alpha i_1 \beta - \psi_1 \beta i_1 \alpha) = \frac{5}{2} p (\psi_2 \alpha i_2 \beta - \psi_2 \beta i_2 \alpha). \]

(14)

By using phasors spatial relationship, the Eq (14) comes into the form:

\[ M_{em} = \frac{5}{2} p \text{Im} (\psi_1 i_1) = \frac{5}{2} p \text{Im} (\psi_2 i_2^*). \]

(15)

We can express the magnetic flux associated with the stator and rotor in terms of inductance and current. The relationship Eq. [18] changes into the following form: [18]

\[ M_{em} = \frac{5}{2} p L_m \text{Im} (i_1 i_1^*). \]

(16)
The calculated time plot of the electromagnetic torque waveform of the five-phase IM pentacle connected and supplied by a rectangular voltage is shown in Fig. 10. The motor is loaded by a nominal load and works at 2850 rev·min\(^{-1}\). In the electromagnetic torque waveform, the fifth harmonic component is well visible. The ripple presents about 20% of the load torque. Torque magnitude is invariant with load change.

5. Conclusion

The article discusses the electromagnetic torque calculation of the five-phase IM which is pentacle connected. The motor is supplied by a five-phase VSI inverter with rectangular output voltage. Mathematical model using the space phasor theory in the complex plane was build.

In the calculated waveform of electromagnetic torque, a huge five harmonic component is visible. This one stays constant with a change in load. It varies only with the shape of the supply voltage.

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References


About Authors

Pavel ZASKALICKY was born in Liptovsky Mikulas (1949), Slovak Republic. He works on the Department of Electrical Engineering and Mechatronics, Faculty of Electrical Engineering and Informatics (FEI) of the Technical University (TU) in Kosice. After graduating from the Faculty of Electrical Engineering of the University of Technology in Kosice (1975) he started to work in Energoprojekt Praha as a designer. Since 1977 he worked as an assistant, later as a lecturer at the Department of Electrical Drives at Faculty of Electrical Engineering of the Technical University of Kosice. In 1985 he received his Ph.D. degree. In 1989 he left for the Universite Technique de Sidi-Bel Abbes, Algeria, where he worked until 1991. In 1991–1995 he worked at Ecole National Superieur d’Electricite et de Mecanique (ENSEM), National Polytechnic Institute of Lorraine (INPL) Nancy France where he was involved both in education and research. He co-authored the general theory of asymmetric reluctance motors.

In 1995 he returned to FEI TU Kosice, where he became Associate Professor in 1997, and in 2007 Full Professor.

Prof. Zaskalicky works in the field of mathematical simulations of electrical machines and power electronic converters. He deals with influence of non-harmonic supply on properties of electrical machines and also with modeling of semiconductor converters using complex Fourier series. He was a successful principal investigator of several research Science Grand Agency (VEGA) and Slovak Research and Development Agency (APVV) projects in Slovak Republic. He is a member of committees of several international conferences and scientific technical journals.

Appendix A

AC Drive Parameters

\[ P_n = 6.5 \text{ kW}; \quad U_n = 5 \times 230 \text{ V}/50 \text{ Hz}; \]
\[ n_n = 2850 \text{ rev} \cdot \text{min}^{-1}; \quad p = 1; \]
\[ R_1 = 3.778 \text{ }\Omega; \quad R'_2 = 2.498 \text{ }\Omega; \]
\[ L_m = 0.436 \text{ H}; \quad L_{1\sigma} = 6.83 \text{ mH}; \quad L_{2\sigma} = 11.88 \text{ mH}; \]