SCANNING OF MARKOVIAN RANDOM PROCESSES
VZORKOVANIE MARKOVOVÝCH NÁHODNÝCH PROCESOV

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Summary Function of most electronic devices is based on the microprocessor control. A microprocessor must scan and evaluate function of many controlled equipment that can be transformed into 2 statuses – "free" or "busy". Microprocessors in a digital exchange are the typical example of that. It can be said the microprocessors are in a discrete dialogue with the controlled parts. They ascertain in regular time intervals whether the status of the controlled equipment has changed. 1 or more "busy" statuses may exist in the same time. As the changes from "free" to "busy" and from "busy" to "free" occur randomly, the state of i busy statuses lasts for a random period too. If the changes have pro-perties of Markovian process, the probability which each change will be detected with and hence the scanning rate needed can be derived.

Abstrakt Činnosť väčšiny elektronických zariadení je založená na mikroprocesorovom riadení. Mikroprocesor musí vzorkovať a vyhodnocovať funkciu mnohých riadených zariadení, ktorá môže byť prevedená do 2 stavov – „voľné“ alebo obsadené“. Typickým príkladom takéhoto riadenia sú mikroprocesory v digitálnych ústredniach. Môžeme povedať, že mikroprocesory vedú prerušovaný dialóg s riadenými časťami. V pravidelných časových odstupoch vyhodnocujú, či sa stav riadeného zariadenia zmenil. V tom istom čase môže byť 1 alebo viacero obsadených stavov. Pretože sa zmeny zo stavu „voľné“ na stav „obsadené“ a opačne vyskytujú náhodne, stav i obsadených zariadení trvá tiež náhodne. Ak tieto zmeny majú vlastnosti Markovovho procesu, je možné odvodiť pravdepodobnosť, s akou bude podchytená každá zmena, a odtiaľ požadovaná hustota vzorkovania.

1. BASIC CONSIDERATIONS

The homogenous (stable in time) Markovian process $X(t)$ with discrete states can be described by these basic equations [1]:

$$P\{X(t+\Delta t)=j/X(t)=i\} = p_{ij}(\Delta t),$$

which is the transition probability of the random process $X(t)$ from the status i which was in the time t to the status j within the time interval $\langle t, t+\Delta t\rangle$, e.g. during the time Δt and

$$P\{X(t+\Delta t)=i/X(t)=i\} = p_{ii}(\Delta t),$$

which is the persistence probability of the random process $X(t)$ in the status i within the time interval $\langle t, t+\Delta t\rangle$, e.g. during the time t.

As the time parameter t is not a discrete value, Δt can generally be 0 and then it would be

$$\lim_{\Delta t\to 0} p_{ij}(\Delta t)=0,$$

$$\lim_{\Delta t\to 0} p_{ii}(\Delta t)=1.$$

Therefore like at probability distributions, it is necessary to use probability densities. Here the transition probability is considered as the ratio

$$\lim_{\Delta t\to 0} \frac{p_{ij}(\Delta t)}{\Delta t} = a_{ij}(t),$$

which is the probability density of the transition of the ran-dom process $X(t)$ from the status i to the status j during the time $\Delta t \to 0$.

As in case of the persistence probability the similar ratio would be:

$$\lim_{\Delta t\to 0} \frac{p_{ii}(\Delta t)}{\Delta t} = \frac{1}{0} = \infty$$

an uncertain infinite small ratio 0/0 shall be used and the persistence probability shall also be transferred to the transition probability

$$q_{ij}(\Delta t) = 1 - p_{ij}(\Delta t) = P\{X(t+\Delta t)\neq iX(t)=i\},$$

where $q_{ij}(\Delta)$ is the leaving probability the status i during the time Δt of the random process $X(t)$. Then

$$\lim_{\Delta t\to 0} \frac{q_{ij}(\Delta t)}{\Delta t} = \lim_{\Delta t\to 0} \frac{1-p_{ii}(\Delta t)}{\Delta t} = a_{ii}(t)$$

is the respective leaving probability density.

As the homogenous Markovian process is assumed, the probability densities are time invariant and it can be put:

$$a_{ij}(t) = a_{ij},$$

$$a_{ii}(t) = a_{ii}.$$

If the time Δt is small enough, it can be assumed that the transition probabilities in the time interval $\langle t, t+\Delta t\rangle$ are proportional to the duration Δt of this interval except of some values of higher orders that would express the probability that more than 1 change occur within this time interval. So it can be put:

$$p_{ij}(\Delta t) = a_{ij}.\Delta t + o(\Delta t), \quad (1)$$

$$q_{ij}(t) = 1 - p_{ii}(\Delta t) = a_{ii}.\Delta t + o(\Delta t), \quad (2)$$

$$\lim_{\Delta t\to 0} \frac{o(\Delta t)}{\Delta t} = 0. \quad (3)$$

The equations (1), (2), (3) express the basic properties of Markovian process [2], [3]. With regard to (3), it can only be:

$$|j-i| = 1.$$

2. MICROSTATES OF MARKOVIAN PROCESS

Let´s examine the random variable T which means the time during that the random process $X(t)$ leaves the status i. Let the distribution function of this random variable be (Fig. 1):

$$F_i(t) = P\{X(\tau\leq t)\neq i/X(t)=i\}.$$

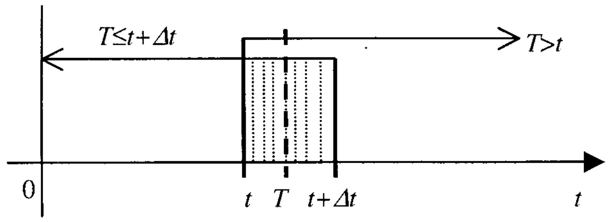

Fig 1. Occurence of the random variable T in the interval ⟨t, t+Δt⟩

Now let's consider that the random variable $T \le t + \Delta t$. This appearance can be composed of 2 disjunctive appearances $T \le t \cup t < T \le t + \Delta t$. The respective probability is:

$$P\{X(T\le t+\Delta t)\ne i/X(T=0)=i\} = P\{X(T\le t)\ne i/X(T=0)=i\} + P\{X(t<T\le t+\Delta t)\ne i/X(T=0)=i\}. \quad (4)$$

According to Figure 1 it can be written:

$$P\{X(t<T\le t+\Delta t)\ne i/X(T=0)=i\} = P\{X(T>t)\ne i/X(T=0)=i\}\times P\{X(T\le t+\Delta t)\ne i/X(T=0)=i\}.$$

If Markovian process is homogenous, it does not depend on the time t. So the time t may be put equal zero and then:

$$P\{X(T\le t+\Delta t)\ne i/X(T=0)=i\} = P\{X(T\le \Delta t)\ne i/X(T=0)=i\} = 1 - p_{ii}(\Delta t)$$

Further:

$$P\{X(T>t)\ne i/X(T=0)=i\} = 1 - P\{X(T\le t)=i/X(T=0)=i\} = F_{ii}(t) = 1 - F_i(t), \quad (5)$$

where $F_{ii}(t)$ is the distribution function of the random variable T which means the time during that the random process $X(t)$ persists in the status i.

Substituting into (4) it is:

$$P\{X(T\le t+\Delta t)\ne i/X(T=0)=i\} = P\{X(T\le t)\ne i/X(T=0)=i\} + P\{X(T>t)\ne i/X(T=0)=i\}.\, P\{X(T\le \Delta t)\ne i/X(T=0)=i\},$$

$$F_i(t+\Delta t) = F_i(t) + [1 - F_i(t)].[1 - p_{ii}(t)]\,,$$

$$\lim_{\Delta t\to 0} \frac{F_i(t+\Delta t) - F_i(t)}{\Delta t} = [1 - F_i(t)] \lim_{\Delta t\to 0} \frac{1 - p_{ii}(\Delta t)}{\Delta t},$$

$$\frac{dF_i(t)}{dt} = [1 - F_{ii}(t)].a_{ii}\,,$$

$$ln[1-F_{ii}(t)] = -a_{ii}t + c,$$

For $t = 0$ is $c = F_i(0) = 0$. Then:

$$F_i(t) = 1 - e^{-a_{ii}t}\,.$$

(6) Further let's examine the random variable T that will now perform the time interval within that just 1 change – the transition from the status i to the status j takes place. 2 disjunctive appearances may occur (Fig. 2): either the random process transits to the status j (the random variable X) or it stays in the status i (the random variable Y). So the random variable T is the sum of 2 random variables X and Y:

$$T = X + Y = \varphi(x,y). \quad (7)$$

The distribution function of the random variable X is given by (1):

$$F_{ij}(x) = a_{ij}.x$$

and the distribution function of the random variable Y is given by (6):

$$F_i(y) = 1 - e^{-a_{ii}y}.$$

The distribution

$$F(t) = P\{T>t\} = P\{\varphi(X,Y)>t\}$$

shall be found.

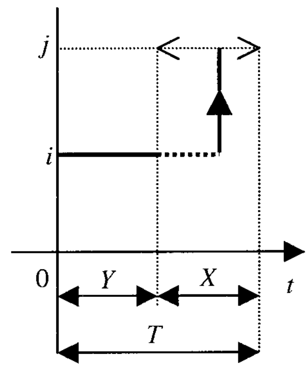

Fig 2. Transition change $i \to j$

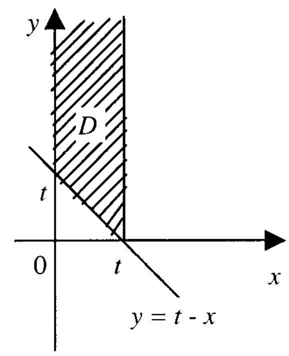

Fig. 3. Definition range of X and Y

Generally, the distribution function consisting of 2 random variables is [4]:

$$F(t) = \iint_D f(x,y)dxdy\,, \quad (8)$$

where $f(x,y)$ is the distribution of the system of 2 random variables X, Y.

The distribution of the random variable X is:

$$f_{ij}(x) = \frac{dF_{ij}(x)}{dx} = a_{ij}$$

and of the random variable Y:

$$f_i(y) = \frac{dF_i(y)}{dy} = a_{ii}.e^{-a_{ii}y}.$$

As the change of the random process and its persistence in the origin status are disjunctive appearances, the random variables X, Y are independent on each other and then

$$f(x,y) = f_{ij}(x).f_i(y) = a_{ij}.a_{ii}.e^{-a_{ii}y}.$$

The integration range is defined by the value t and by the function (7) (Fig. 3). Then in accordance with (8) is:

$$F(t) = \iint_D f_{ij}(x).f_i(y)dxdy = \int_0^t\left(\int_{t-x}^{\infty} a_{ij}a_{ii}e^{-a_{ii}y}dy\right)dx =$$

$$= a_{ij}\int_0^t e^{-a_{ii}(t-x)}dx = a_{ij}e^{-a_{ii}t}\int_0^t e^{a_{ii}x}dx = \frac{a_{ij}}{a_{ii}}(1 - e^{-a_{ii}t}).$$

The distribution of the random variable T is:

$$f(t) = \frac{dF(t)}{dt} = a_{ij}.e^{-a_{ii}t}. \qquad (9)$$

If $t \to \infty$ then

$$F(\infty) = \frac{a_{ij}}{a_{ii}}, \qquad (10)$$

which is the transition probability of Markovian process from the status i to the status j in the stable state.

Similarly, the next random variable T may be examined that will perform the time interval within that just 1 transition from the status i to the status j occurs and the random process will stay in the new status j after the change has taken place (Fig. 4).

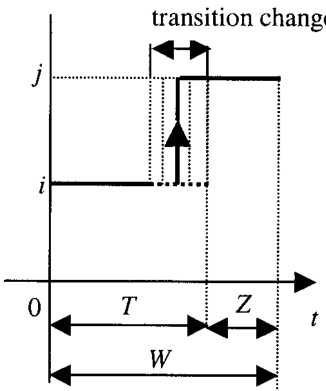

Fig. 4. Transition and stay in new status

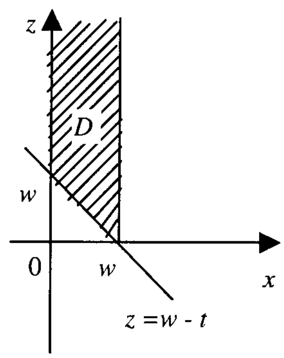

Fig. 5. Definition range of T and W

Again, the random variable W is the sum of 2 independent random variables T and Z:

$W = T + Z = \varphi(T,Z)$.

The distribution of the random variable T is given by (9). According to (6), the distribution function of the random variable Z is:

$$F_j(z) = 1 - e^{-a_{jj}z}$$

and the distribution:

$$f_j(z) = \frac{dF_j(z)}{dz} = a_{jj}.e^{-a_{jj}z}.$$

Then

$$f(t,z) = f(t).f_j(z) = a_{ij}.a_{jj}.e^{-a_{ii}t}.e^{-a_{jj}z}.$$

The calculation in accordance with (8) and Fig. 5 gives:

$$F(w) = \iint_D f(t).f_j(z)dtdz = \int_0^w \int_{w-t}^{\infty} a_{ij}a_{jj}e^{-a_{ii}t}e^{-a_{jj}z}dtdz =$$

$$= a_{ij}a_{jj}\int_0^w \left(e^{-a_{ii}t} \int_{w-t}^{\infty} e^{-a_{jj}z}dz \right) dt = a_{ij}\int_0^w e^{-a_{ii}t}e^{-a_{jj}(w-t)}dt =$$

$$= a_{ij}e^{-a_{jj}w}\int_0^w e^{-(a_{ii}-a_{jj})t}dt = a_{ij}e^{-a_{jj}w}.\frac{1-e^{-(a_{ii}-a_{jj})w}}{a_{ii}-a_{jj}} =$$

$$= \frac{a_{ij}}{a_{ii}-a_{jj}}\left(e^{-a_{jj}w} - e^{-a_{ii}w}\right). \qquad (11)$$

3. SCANNING PROCESS

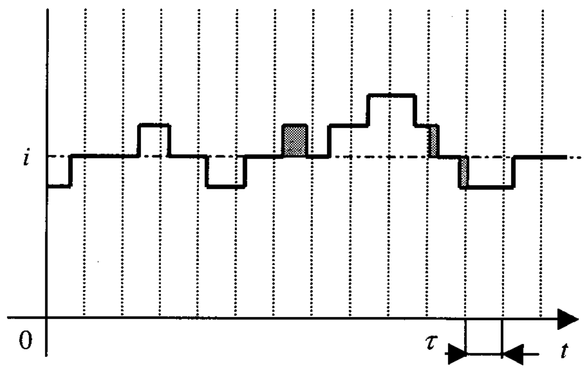

Fig. 6. Scanning procedure

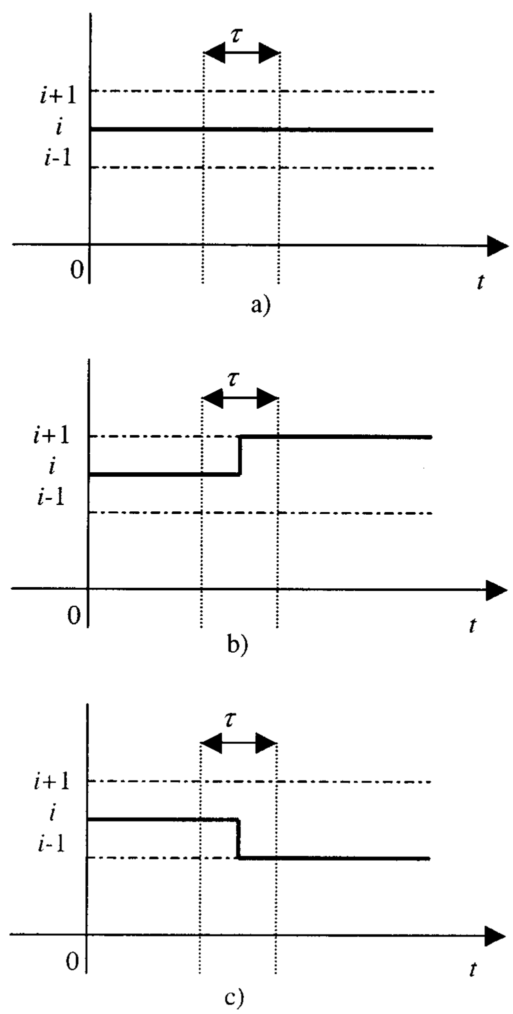

Fig 7. Allowed changes during scanning interval τ

Figure 6 gives an example of Markovian process with discrete states which is scanned at the regular time intervals of τ. The goal is to catch up as many changes as possible during scanning procedure. Of course, due to the random property of Markovian process all changes may be caught up only with a certain probability. Should all changes be caught up with the probability 1, the scanning interval would have to be infinitely short.

In order to catch up each change in the state of the random process, only the next 3 causes are allowed during any scanning interval τ:

- no change occurs – the process remains in the status i (Fig. 7a) or
- just 1 change occurs – the transition from the status i to the higher status $j = i + 1$ (Fig. 7b) or
- just 1 change occurs – the transition from the status i to lower status $j = i - 1$ (Fig. 7c).

The respective probabilities are given by (5), (6) and (11):

$$p_{ii}(T>\tau) = 1 - F_i(\tau) = e^{-a_{ii}\tau}, \qquad (12)$$

$$p_{i\,i+1}(T>\tau) = F(\tau) = \frac{a_{i\,i+1}}{a_{ii} - a_{i+1\,i+1}}\left(e^{-a_{i+1\,i+1}\tau} - e^{-a_{ii}\tau}\right) \qquad (13)$$

$$p_{i\,i-1}(T>\tau) = F(\tau) = \frac{a_{i\,i-1}}{a_{ii} - a_{i-1\,i-1}}\left(e^{-a_{i-1\,i-1}\tau} - e^{-a_{ii}\tau}\right) \qquad (14)$$

Now it is necessary to find the transition probability densities a_{ii}, $a_{i\,i+1}$, $a_{i\,i-1}$, a_{i+1}, $a_{i\,i-1}$.

Let λ be defined as the count of transitions from the free to the busy status of a non active scanned equipment and μ as the count of transitions from the busy to the free status of an active scanned equipment within a time unit where N denotes the number of scanned equipment. The ratio

$$a = \frac{\lambda}{\mu} \qquad (15)$$

is traffic load offered by 1 equipment ($0 < a < 1$) and

$A = a.N$

is the total offered traffic load.

The random process remains in the status i when neither the transition to the higher status i+1 nor the transition to the lower status i-1 takes place during the time interval Δt:

$$p_{ii}(\Delta t) = (1 - \lambda_i\Delta t).(1 - \mu_i\Delta t) = [1 - (N-i)\lambda\Delta t].(1 - i\mu\Delta t) =$$
$$1 - i\mu\Delta t - (N-i)\lambda\Delta t + (N-i)i\lambda\mu(\Delta t)^2 \approx$$
$$\approx 1 - [(N-i)\lambda + i\mu]\Delta t. \qquad (16)$$

The expression of higher order $(\Delta t)^2$ may be neglected.

The random process transits from the status i to the higher status $j = i + 1$ when just 1 transition to the higher status $j = i + 1$ and no transition to the lower status $j = i - 1$ takes place during the time interval Δt:

$$p_{i\,i+1}(\Delta t) = \lambda_i\Delta t.(1 - \mu_i\Delta t) = (N-i)\lambda\Delta t.(1 - i\mu\Delta t) =$$
$$= (N-i)\lambda\Delta t - (N-i)i\lambda\mu(\Delta t)^2 \approx (N-i)\lambda\Delta t. \qquad (17)$$

The random process transits from the status i to the lower status $j = i - 1$ when just 1 transition to the lower status $j = i - 1$ and no transition to the higher status $j = i + 1$ takes place during the time interval Δt:

$$p_{i\,i-1}(\Delta t) = \mu_i\Delta t.(1 - \lambda_i\Delta t) = i\mu\Delta t.[1 - (N-i)\lambda\Delta t] =$$
$$= i\mu\Delta t - (N-i)i\lambda\mu(\Delta t)^2 \approx i\mu\Delta t. \qquad (18)$$

Comparing (16), (17), (18) to (2) and (1) we have:

$$a_{ii} = (N-i)\lambda + i\mu, \qquad (19)$$

$$a_{i\,i+1} = (N-i)\lambda, \qquad (20)$$

$$a_{i\,i-1} = i\mu \qquad (21)$$

$$a_{jj} = \begin{cases} a_{i+1\,i+1} = (N-i-1)\lambda + (i+1)\mu \\ a_{i-1\,i-1} = (N-i+1)\lambda + (i-1)\mu \end{cases}. \qquad (22)$$

Substitution (19), (20), (21), (22) into (12), (13), (14) gives:

$$p_{ii}(T>\tau) = e^{-[(N-i)\lambda + i\mu]\tau} = e^{-[a(N-i)+i]x},$$

$$p_{i\,i+1}(T>\tau) = \frac{(N-i)\lambda}{(N-i)\lambda + i\mu - [(N-i-1)\lambda + (i+1)\mu]} \times$$
$$\times\left\{e^{-[(N-i-1)\lambda+(i+1)\mu]\tau} - e^{-[(N-i)\lambda+i\mu]\tau}\right\} =$$
$$= \frac{(N-i)\lambda}{\lambda - \mu}\left\{e^{-[(N-i-1)\lambda+(i+1)\mu]\tau} - e^{-[(N-i)\lambda+i\mu]\tau}\right\} =$$
$$= \frac{a(N-i)}{a-1}\left\{e^{-[a(N-i-1)+i+1]x} - e^{-[a(N-i)+i]x}\right\},$$

$$p_{i\,i-1}(T>\tau) = \frac{i\mu}{(N-i)\lambda + i\mu - [(N-i+1)\lambda + (i-1)\mu]} \times$$
$$\times\left\{e^{-[(N-i+1)\lambda+(i-1)\mu]\tau} - e^{-[(N-i)\lambda+i\mu]\tau}\right\} =$$
$$= \frac{i\mu}{\lambda - \mu}\left\{e^{-[(N-i)\lambda+i\mu]\tau} - e^{-[(N-i+1)\lambda+(i-1)\mu]\tau}\right\} =$$
$$= \frac{i}{a-1}\left\{e^{-[a(N-i)+i]x} - e^{-[a(N-i+1)+i-1]x}\right\}.$$

Here a is given by (15) and

$$x = \tau.\mu = \frac{\tau}{\bar{t}},$$

where $\bar{t}$ is the average duration of a busy status (mean holding time).

The probability that maximum 1 change occurs within the scanning interval τ that is also the probability that each change will be caught up within this interval is:

$$P\{T>\tau\} = p_{ii}(T>\tau) + p_{i\,i+1}(T>\tau) + p_{i\,i-1}(T>\tau) =$$

$$e^{-[a(N-i)+i]x} + \frac{a(N-i)}{a-1}\left\{e^{-[a(N-i-1)+i+1]x} - e^{-[a(N-i)+i]x}\right\} +$$

$$+\frac{i}{a-1}\left\{e^{-[a(N-i)+i]x} - e^{-[a(N-i+1)+i-1]x}\right\} =$$

$$=\frac{1}{a-1}\left\{-[a(N-i-1)-i+1].e^{-[a(N-i)+i]x} +\right.$$

$$\left.+a(N-i).e^{-[a(N-i-1)+i+1]x} - i.e^{-[a(N-i+1)+i-1]x}\right\} = P\{X>x\}.$$

The distribution function of the random variable X is:

$$F(x) = P\{X \le x\} = 1 - P\{X>x\}$$

and the probability distribution:

$$f(x) = \frac{dF(x)}{dx} = \frac{d}{dx}\left[1 - P\{X>x\}\right] =$$

$$=\frac{1}{a-1}\left\{-[a(N-i-1)-i+1].[a(N-i)+i].e^{-[a(N-i)+i]x} +\right.$$

$$+a(N-i).[a(N-i-1)+i+1].e^{-[a(N-i-1)+i+1]x} -$$

$$\left.-i.[a(N-i+1)+i-1].e^{-[a(N-i+1)+i-1]x}\right\}.$$

As $f(x)$ is the probability distribution, it must be:

$$\int_0^{\infty} f(x)dx = 1,$$

which is fulfilled.

The mean of the random variable X is:

$$\bar{x} = \int_0^{\infty} x.f(x)dx = \frac{1}{a-1}\left[-\frac{a(N-i-1)-i+1}{a(N-i)+i} + \right.$$

$$\left.+\frac{a(N-i)}{a(N-i-1)+i+1} - \frac{i}{a(N-i+1)+i-1}\right]. \quad (23)$$

Let the count of busy statuses be observed during a time Θ. Let n_1 busy statuses were during t_1, n_2 busy statuses during t_2,..., t_m busy statuses during t_m whereby $t_1 + t_2 + ... + t_m = \Theta$. The total time of all observed busy statuses is $n_1 t_1 + n_2 t_2 + ... + n_m t_m$. As traffic load is the total busy time related to the observation time, it can be written:

$$A = a.N = \lim_{\Theta\to\infty} \frac{1}{\Theta}\sum_{k=1}^{m} i_k t_k = \lim_{\Theta\to\infty}\sum_{k=1}^{m} i_k.\frac{t_k}{\Theta} = \sum_{k=1}^{m} i_k p_k = \bar{i}, \quad (24)$$

where $\bar{i}$ is the mean count of busy statuses. Substituting the closest integer value of $\bar{i} = a.N$ instead of i in (23) the mean value $\bar{x}$ as function of a and N may be calculated (Fig. 8).

If the number of controlled devices is very large like at local exchanges, $N >> i$ and i may be neglected with regard to N. The (19), (20), (21), (22) will be:

$$a_{ii} = N.\lambda + i\mu = \mu\left(N.\frac{\lambda}{\mu} + i\right) = \frac{N.a + i}{\bar{t}} = \frac{A+i}{\bar{t}},$$

$$a_{i\,i+1} = N.\lambda = N.\frac{\lambda}{\mu}.\mu = N.a.\frac{1}{\bar{t}} = \frac{A}{\bar{t}},$$

$$a_{i\,i-1} = i\mu = \frac{i}{\bar{t}},$$

$$a_{jj} = \begin{cases} a_{i+1\,i+1} = N.\lambda + (i+1)\mu = \mu\left(N.\frac{\lambda}{\mu} + i + 1\right) = \frac{A+i+1}{\bar{t}} \\ a_{i-1\,i-1} = N.\lambda + (i-1)\mu = \mu\left(N.\frac{\lambda}{\mu} + i - 1\right) = \frac{A+i-1}{\bar{t}} \end{cases}$$

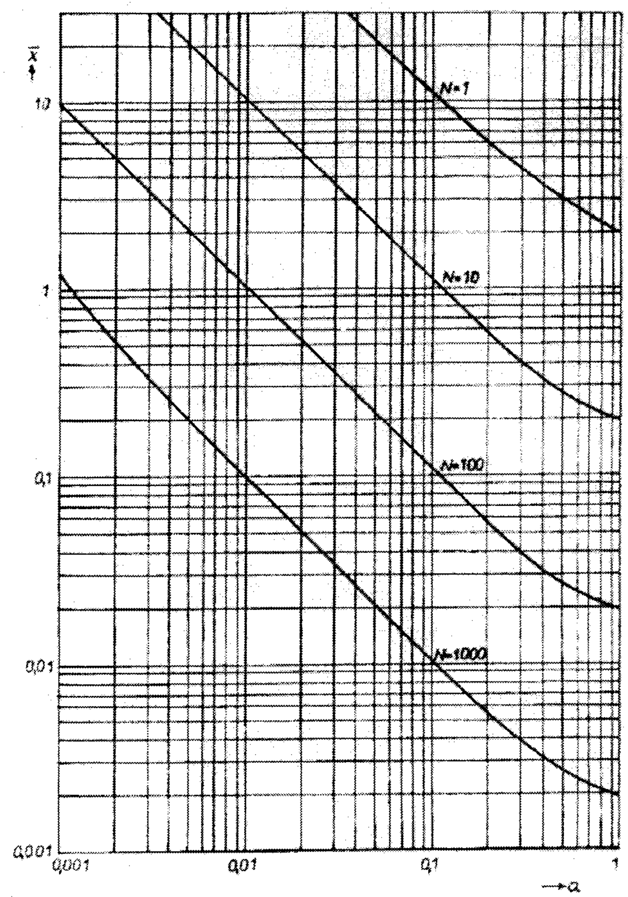

Fig. 8. Mean value of x versus a with N as parameter

$$f(x) = \frac{d}{dx}\left(1 - P\{X>x\}\right) =$$

$$=\frac{d}{dx}\left\{1 - \left[p_{ii}(X>x) + p_{i\,i+1}(X>x) + p_{i\,i-1}(X>x)\right]\right\} =$$

$$=\frac{d}{dx}\left(1 - \left\{e^{-(A+i)x} + \right.\right.$$

$$+\frac{A}{A+i-(A+i+1)}\left[e^{-(A+i+1)x} - e^{-(A+i)x}\right] +$$

$$\left.\left.+\frac{i}{A+i-(A+i-1)}\left[e^{-(A+i-1)x} - e^{-(A+i)x}\right]\right\}\right) =$$

$$= \frac{d}{dx}\left\{1 - \left[(A-i+1)e^{-(A+i)x} - Ae^{-(A+i+1)x} + ie^{-(A+i-1)x}\right]\right\} =$$

$$= (A-i+1).(A+i)e^{-(A+i)x} - A.(A+i+1)e^{-(A+i+1)x} +$$

$$+ i.(A+i-1)e^{-(A+i-1)x}.$$

The mean value of this distribution is:

$$\bar{x} = \frac{A-i+1}{A+i} - \frac{A}{A+i+1} + \frac{i}{A+i-1}.$$

Assuming $i = A$ according to (24) and neglecting 1 with regard to $A + i$ we have:

$$\bar{x} = \frac{\bar{\tau}}{\bar{t}} \approx \frac{1}{2A} - \frac{A}{2A} + \frac{A}{2A} = \frac{1}{2A} \qquad (25)$$

and the average scanning interval will be:

$$\bar{\tau} \approx \bar{x}.\bar{t} = \frac{\bar{t}}{2A}.$$

4. CONCLUSION

Comparing (25) to the Channon formula

$$\tau = \frac{1}{2f_{max}}$$

we find that these formulae are the same in terms of quality. The upper frequency f_{max} of a sampled signal is exactly known and so the corresponding sampling interval τ may be determined sharp. Both f_{max} and τ are common variables unlike of relative scanning interval x and traffic A that are random variables. Therefore the scanning interval $\bar{\tau}$ can not be determined exactly but only as a mean value. By means of comparison of these 2 formulae it can be seen a tight connection between 2 different science theories – theory of signal processing and theory of random processes.

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