Abstract. This paper applies a relatively new optimization method, the Grey Wolf Optimizer (GWO) algorithm for Optimal Power Flow (OPF) of two-terminal High Voltage Direct Current (HVDC) electrical power system. The OPF problem of pure AC power systems considers the minimization of total costs under equality and inequality constraints. Hence, the OPF problem of integrated AC-DC power systems is extended to incorporate HVDC links, while taking into consideration the power transfer control characteristics using a GWO algorithm. This algorithm is inspired by the hunting behavior and social leadership of grey wolves in nature. The proposed algorithm is applied to two different case-studies: the modified 5-bus and WSCC 9-bus test systems. The validity of the proposed algorithm is demonstrated by comparing the obtained results with those reported in literature using other optimization techniques. Analysis of the obtained results show that the proposed GWO algorithm is able to achieve shorter CPU time, as well as minimized total cost when compared with already existing optimization techniques. This conclusion proves the efficiency of the GWO algorithm.

1. Introduction

In a High Voltage Direct Current (HVDC) transmission system, an inverter station converts the AC electrical power into DC. After transmission, a rectifier converts the DC electrical power back to AC. These converters can be located in one place as a back-to-back HVDC system, or electrical power can be transmitted from one converter station to another over long distance via an overhead transmission line or an underground cable [1]. HVDC systems serve as ideal supplements to existing AC power networks. The advantages of using HVDC systems include providing economical and more efficient transmission of electrical power over long distances, solving synchronism-related problems by connecting asynchronous networks or networks which operate at different frequencies, providing controlled power supply in either direction and offering access for onshore and offshore power generation from renewable energy sources [2].

As reported in literature, the first commercial application of HVDC transmission took place between the Swedish mainland and the island of Gotland in 1954, using mercury-arc valves. The first 320 MW, thyristor-based HVDC system was commissioned in 1972 between Canadian provinces of New-Brunswick and Quebec [3]. The converters used for HVDC systems are grouped into these categories: line-commutated converters and voltage-source converters or current-source converters. In AC power systems, the Optimal Power Flow (OPF) problem is defined by nonlinear, non-convex equations.

Feasibility studies are required to determine preliminary parameters of the planned system modifications. To incorporate results of power flow and other parame-

Keywords

Grey Wolf Optimizer (GWO) algorithm, High Voltage Direct Current (HVDC), intelligent power systems, Optimal Power Flow (OPF), optimization methods.
ters, more detailed studies are needed. Finally, operating studies are necessary to successfully integrate the HVDC facility into the power system [4]. In HVDC systems, where no reactive power is involved, the OPF problem is less complex but it still retains its nonlinear characteristic when voltage control and optimal storage operation are included in the formulation. There are several different methods solving the resulting nonlinear equations [5].

More recently, some authors have proposed other methods to solve this problem. Authors in [6] and [7] present the model of a voltage-source converter suitable for OPF solution of HVDC using Newton Raphson Algorithm (NRA) and a sequential method was introduced [8]. A new approach for load flow analysis of integrated HVDC power systems using sequential modified Gauss-Seidel method was reported [9]. In [10], a multi-terminal HVDC power flow with a conventional AC power flow has been proposed. In [11], a steady-state multi-terminal HVDC model for power flow has been developed and it includes converter limits, as well as different converter topologies. Other authors have solved this problem by applying new techniques, such as Artificial Bee Colony (ABC) algorithm [12], Genetic Algorithm (GA) [13] and Backtracking Search Algorithm (BSA) [14]. Authors in [15] proposed an OPF in order to minimize the losses in a multi-terminal HVDC grid. Application of transient stability constraints for OPF, to a transmission system including an HVDC, was proposed [16]. Authors in [17] applied an informed gap decision theory to the OPF model for the optimal operation of AC-DC systems with offshore wind farms.

In the last two years, Grey Wolf Optimizer (GWO) algorithm has been applied in power systems for solving combined economic emission dispatch problems [18]. In [19], studying the blackout risk prevention in a smart grid based flexible optimal strategy [19] and estimating the parameters of the Proportional Integral (PI) controller for automatic generation control of two-area power system [20]. Furthermore, it has been used for optimizing wide-area power system stabilizer design [21], solving OPF problem [22] and [23], load frequency control of interconnected power system [24] and economic dispatch problems [25]. It has been also used for solving the optimizing PID controller for automatic generation control of a multi-area thermal power system [26] and the design of Static Synchronous Series Compensator (SSSC) based stabilizer to damp inter-area oscillations [27].

In this paper, the GWO algorithm is used to achieve OPF of the two-terminal HVDC system. The proposed algorithm is applied to two different case-studies which are: the modified 5-bus andWSCC 9-bus test systems. The validity and efficiency of the proposed algorithm are evaluated by comparing the obtained results with those obtained when applying other methods reported in literature such as Backtracking Search Algorithm (BSA) [14], Artificial Bee Colony (ABC) algorithm [12], Genetic Algorithm (GA) [13] and Newton-Raphson Method (NRM) [7].

2. Two-Terminal HVDC Modeling

A basic schematic diagram of a two-terminal HVDC transmission link is given in Fig. 1.

![Fig. 1: A basic schematic diagram of a two-terminal HVDC transmission link.](image)

In Fig. 1, $v_r$ and $v_i$ are the AC voltages (rms) at the converter transformer primary, $i_r$ and $i_i$ are the currents at the AC sides of the rectifier and inverter. $\delta_r$ and $\delta_i$ are the bus voltage phase angles, $\xi_r$ and $\xi_i$ are the current angles and $r_{dc}$ is DC link resistance. $p_r$ and $p_i$ are the active powers at the rectifier and inverter sides, $q_r$ and $q_i$ are the reactive powers at the rectifier and inverter sides and $i_{id}$ is the direct current of HVDC link. The basic converter equations between AC and DC sides for the rectifier terminal are expressed as follows [7], [12], [13] and [14]:

\begin{align}
    v_{dor} &= k \cdot t_r \cdot v_r, \quad (1) \\
    v_{dr} &= v_{dor} \cos \alpha - r_{cr} \cdot i_{id}, \quad (2) \\
    p_r &= v_{dr} \cdot i_{id}, \quad (3) \\
    \phi_r &= \cos^{-1} \left( \frac{v_{dr}}{v_{dor}} \right), \quad (4) \\
    q_r &= |p_r \cdot \tan \phi_r|, \quad (5)
\end{align}

where the constant $k$ is equal to $3\sqrt{2}/\pi$, $v_{dor}$ is the open circuit DC voltage at the rectifier side; $r_{cr}$ is the equivalent commutation resistance at the rectifier side and equal to $\sqrt{3}x_{cr}/\pi$ ($x_{cr}$ is the equivalent commutation reactance at the rectifier side), and $\phi_r = \delta_r - \xi_r$ is the phase angle between the AC voltage and the fundamental AC current at the rectifier side.
The basic converter equations between AC and DC sides for the inverter terminal are also expressed as follows [7, 12, 13] and [14]:

\[ v_{doi} = k \cdot t_i \cdot v_i, \]
\[ v_{di} = v_{doi} \cos \gamma - rc_i \cdot i_d, \]
\[ p_i = v_{di} \cdot i_d, \]
\[ \phi_i = \cos^{-1} \left( \frac{v_{di}}{v_{doi}} \right), \]
\[ q_i = |p_i \cdot \tan \phi_i|, \]

where \( v_{doi} \) is the open circuit dc voltage at the inverter side; \( rc_i \) is the equivalent commutation resistance at the inverter side, and \( \Phi_i = \delta_i - \xi_i \) is the phase angle between the AC voltage and the fundamental AC current at the inverter side. An equivalent circuit of a two-terminal HVDC link is shown in Fig. 2.

![Fig. 2: An equivalent circuit of a two-terminal HVDC link.](image)

In Fig. 2, \( \alpha \) is the ignition delay angle, \( \gamma \) is the extinction advance angle, and \( v_{dr} \) and \( v_{di} \) are the DC link voltages at the rectifier and inverter terminals. The relationship between the rectifier and inverter terminal voltages of the DC link can be expressed by considering DC link resistance as follows:

\[ v_{dr} - v_{di} - r_{dc} \cdot i_d = 0. \]

### 3. Problem Formulation and Constraints

The OPF is an optimization problem whose mathematical equations are expressed as follows:

Minimize \( f(x, u) \). \hspace{1cm} (12)

Subject to:
\[
\begin{align*}
 g(x, u) &= 0, \\
 h(x, u) &\leq 0.
\end{align*} \hspace{1cm} (13)
\]

The objective function \( f(x, u) \) considers the production cost of the entire power system and the equality constraints \( g(x, u) \) consider the power flow equations related to the entire power system. The inequality constraints \( h(x, u) \) consider the limits of the variables related to the entire power system. The variables \( x = (x_1, \ldots, x_n) \) and \( u = (u_1, \ldots, u_n) \) of these functions are the state and control vectors, respectively.

### 3.1. Control Variables

The control variables should be the same as those of the problem to be optimized. The AC and DC system state variables in per unit are selected as follows [12, 13] and [14]:

\[ x = [x_{AC}, x_{DC}], \]
\[ u = [u_{AC}, u_{DC}], \]

where \( p_{gslack}, q_{g}, v_{g}, \alpha, \gamma, v_{dr}, v_{di}, \) \( p_{gi} \) is the slack bus active power output, \( q_{gi} \) is the reactive power outputs, \( v_{rs} \) is the load bus voltage magnitudes, and \( N_l \) is the number of load buses. The AC and DC system control variables in per unit are also selected as follows [12, 13] and [14]:

\[ x_{AC} = [p_{g1}, \ldots, p_{gN_g}, v_{Li}, \ldots, v_{LN}], \]
\[ x_{DC} = [t_r, t_i, \alpha, \gamma, v_{dr}, v_{di}], \]

where \( p_{g1} \) (except for the slack bus \( p_{gslack} \)) is the generator active power outputs, \( v_{gi} \) is the generator voltage magnitudes, \( N_g \) is the number of the generator buses, \( t_r \) is the transformer tap ratios, \( N_T \) is the number of transformers, \( p_r \) and \( p_i \) are the active powers at the rectifier and inverter sides, \( q_r \) and \( q_i \) are the reactive powers at the rectifier and inverter sides, and \( i_d \) is the direct current of HVDC link.

### 3.2. Objective Function

The problem is minimization of the total production cost \( F_{\text{cost}} \) in a power system. In other words, the aim is minimization of objective function which is power loss in an energy system. At the same time, the objective function of whole system is minimized under equality and inequality constraints. Therefore, the objective function \( f \) can be as follows:

\[ f(x, u) = F_{\text{cost}} = \sum_{i=1}^{N_g} (a_i \cdot p_{gi}^2 + b_i \cdot p_{gi} + c_i), \]

where \( F_{\text{cost}} \) represents the total production cost; \( a_i, b_i \) and \( c_i \) represent the production cost coefficients of the \( i^{\text{th}} \) generator.
3.3. Equality Constraints

1) AC System Equality Constraints

A representation of the AC bus connected to the DC transmission link is shown in Fig. 3. The equalities related to the $k^{th}$ bus are given by:

$$p_{gk} - p_{r} - p_{dk} - p_{k} = 0,$$

$$q_{gk} + q_{dk} - q_{r} - q_{k} = 0.$$

(21)  

(22)

In Fig. 3, $p$, $q$, $v$ and $\delta$ represent the active power, reactive power, bus voltage magnitudes, and bus voltage angles, respectively. The subscripts $g$, $l$, $s$ and $d$ represent generator, load, shunt reactive compensator, and DC link, respectively.

For stable operation, the values of the generator active and reactive power outputs, bus voltage magnitudes, transformer tap ratios and shunt VAR compensation are restricted by their lower and upper limits as follows [7], [12], [13] and [14]:

$$p_{gi}^{\min} \leq p_{gi} \leq p_{gi}^{\max} \quad i = 1, \ldots, N_g,$$

$$q_{gi}^{\min} \leq q_{gi} \leq q_{gi}^{\max} \quad i = 1, \ldots, N_g,$$

$$q_{si}^{\min} \leq q_{si} \leq q_{si}^{\max} \quad i = 1, \ldots, N_s,$$

$$v_{i}^{\min} \leq v_{i} \leq v_{i}^{\max} \quad i = 1, \ldots, N,$$

$$t_{i}^{\min} \leq t_{i} \leq t_{i}^{\max} \quad i = 1, \ldots, N_T,$$

where $N_c$ is the number of the compensation devices.

3.4. Generation Capacity Constraints

These constraints are represented by the upper and lower limits of the corresponding variables as follows [7], [12], [13] and [14]:

$$p_{dk}^{\min} \leq p_{dk} \leq p_{dk}^{\max} \quad k = 1, 2,$$

$$q_{dk}^{\min} \leq q_{dk} \leq q_{dk}^{\max} \quad k = 1, 2,$$

$$v_{dk}^{\min} \leq v_{dk} \leq v_{dk}^{\max} \quad k = 1, 2,$$

$$t_{dk}^{\min} \leq t_{dk} \leq t_{dk}^{\max} \quad k = 1, 2,$$

$$i_{d}^{\min} \leq i_{d} \leq i_{d}^{\max},$$

$$\alpha^{\min} \leq \alpha \leq \alpha^{\max},$$

$$\gamma^{\min} \leq \gamma \leq \gamma^{\max}.$$
4. Grey Wolf Optimizer (GWO) Algorithm

The Grey Wolf Optimizer (GWO) algorithm mimics the leadership hierarchy and hunting mechanism of grey wolves in nature proposed by Mirjalili et al. in 2014 [28] and [29]. The hunting technique and the social hierarchy of grey wolves are mathematically modeled in order to design GWO and perform optimization.

The mathematical model of the encircling behavior is represented by the following equations [28]:

\[
\vec{D} = |\vec{C} \cdot \vec{X}_p(\text{iter}) - \vec{X}(\text{iter})|,
\]

\[
\vec{X}_{\text{iter} + 1} = \vec{X}_p(\text{iter}) - \vec{A} \cdot \vec{D},
\]

where iter is the current iteration and \(X_P(\text{iter})\) represents the position vector of the victim. The \(\vec{A}\) and \(\vec{C}\) are coefficient vectors which are given by:

\[
\begin{align*}
\vec{A} &= 2 \cdot \vec{a} \cdot \vec{r}_1 - \vec{a}, \\
\vec{C} &= 2 \cdot \vec{r}_2.
\end{align*}
\]

where \(\vec{a}\) is linearly decreased from 2 to 0 over the course of iterations, \(\vec{r}_1\) and \(\vec{r}_2\) are random vectors in the range \([0, 1]\).

In GWO, the first three best solutions obtained are stored so far and push the other search agents to update their positions due to the position of the best search agents. In order to formulate the social hierarchy of wolves when designing GWO algorithm, the population is split into four groups: alpha (\(\alpha\)), beta (\(\beta\)), delta (\(\delta\)) and omega (\(\omega\)).

Over the course of iterations, the first three best solutions are called \(\alpha\) and \(\delta\), respectively. Figure 4 shows how to update the locations of \(\alpha\), \(\beta\) and \(\delta\), respectively, in a two-dimensional space.

Fig. 4: Position updating in GWO [28]

The rest of the candidate solutions are denoted as \(\omega\). In this algorithm, the hunting/optimization is guided by \(\alpha\), \(\beta\), \(\delta\) and \(\omega\). The wolves are required to encircle \(\alpha\) and \(\delta\) to find better solutions [28, 29] and [30].

Save the first three best solutions obtained so far and oblige the other search agents (including the omegas) to update their positions according to the position of the best search agent. The following formulas are proposed in this regard.

\[
\vec{D}_\alpha = |\vec{C}_1 \cdot \vec{X}_\alpha - \vec{X}|,
\]

\[
\vec{D}_\beta = |\vec{C}_2 \cdot \vec{X}_\beta - \vec{X}|,
\]

\[
\vec{D}_\delta = |\vec{C}_3 \cdot \vec{X}_\delta - \vec{X}|,
\]

\[
\vec{X}_1 = \vec{X}_\alpha - \vec{A}_1 \cdot (\vec{D}_\alpha),
\]

\[
\vec{X}_2 = \vec{X}_\beta - \vec{A}_2 \cdot (\vec{D}_\beta),
\]

\[
\vec{X}_3 = \vec{X}_\delta - \vec{A}_3 \cdot (\vec{D}_\delta),
\]

\[
\vec{X}(\text{iter} + 1) = \frac{\vec{X}_1 + \vec{X}_2 + \vec{X}_3}{3}.
\]

With these equations, a search agent updates its position according to \(\alpha\), \(\beta\), \(\delta\), in a dimensional search space.

In these formulas, vectors \(\vec{A}\) and \(\vec{C}\) are obliging the GWO algorithm to explore and exploit the search space. With decreasing \(\vec{A}\), half of the iterations is devoted to exploration (\(|\vec{A}| \geq 1|\)) and the other half is dedicated to exploitation (\(|\vec{A}| < 1|\)).

The range of \(\vec{C}\) is \(2 \leq \vec{C} \leq 0\) and the vector \(\vec{C}\) also improves exploration when \(\vec{C} > 1\). Exploration is emphasized when \(\vec{C} < 1\); \(\vec{A}\) is decreased linearly over the course of the iterations [29] and [30]. In contrast, \(\vec{C}\) is generated randomly to emphasize the exploration/exploitation at any stage and to avoid local optima.

5. Simulation Results

To show the applicability and efficiency of the proposed GWO optimization algorithm in solving the OPF problem of a two-terminal HVDC system, we tested it on the two test systems. The parameters used for the proposed GWO algorithm are given by: the population size is 80, the coefficient \(a\) is between \([0, 2]\), the random vectors \(\vec{r}_1\) and \(\vec{r}_2\) belong to interval \([0, 1]\) and the stopping criterion of the algorithm is set as 100 iterations for each of the two test systems. The developed
program using MATLAB is run on a computer with processor Intel-core i3-5010 U, CPU 2.1 GHz, 4 GB RAM.

Figure 5 shows the flow chart of the sequential power flow algorithm now combined in a sequential AC/DC [11] and [31]. Applied the traditional Newton-Raphson method is used for the sequential AC power flow algorithm is the first step and a linear current-balancing method is used for the sequential DC power flow algorithm.

![Flow chart of the sequential VSC AC/DC power flow algorithm](image)

Fig. 5: Flow chart of the sequential VSC AC/DC power flow algorithm [11] and [31].

5.1. First Case-Study: 5-Bus Test System

The system shown in Fig. 6 [7] has five buses and two generators and it is extended with a two-terminal HVDC link. The AC network and HVDC converters are assumed to work under three-phase balanced conditions. The experiment is performed for two different scenarios, according to the power and current of the DC link, which are:

- Scenario A: The current of the DC link is considered to be 0.10 p.u.
- Scenario B: The current of the DC link is considered to be 0.15 p.u.

![The 5-bus test system](image)

Fig. 6: The 5-bus test system.

The convergence characteristics of the GWO optimization algorithm for the two scenarios of the first case-study are presented in Fig. 7. A lower DC current, as given in Scenario A, encounters less iterations than for Scenario B.

![Convergence curve of the GWO algorithm for the two scenarios of the first case-study](image)

Fig. 7: Convergence curve of the GWO algorithm for the two scenarios of the first case-study.

Table 1 and Tab. 2 represent the simulation results obtained when applying BSA [14], ABC [12], GA [13], NRM [7], and GWO algorithms to the two scenarios of the first case-study.

5.2. Second-Case Study: WSCC 9-Bus Test System

The WSCC 9-bus system shown in Fig. 8 [13] consists of three generators, six transmission line, three power transformers, and three loads connected at buses 5, 6,
Tab. 1: The first case-study (Scenario A): Simulation results obtained for BSA, ABC, GA, NRM and GWO algorithms.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Limit (p.u.)</th>
<th>BSA</th>
<th>ABC</th>
<th>GA</th>
<th>NRM</th>
<th>GWO</th>
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<td>Min</td>
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<td>Generator Active Outputs</td>
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<tr>
<td>$p_{g1}$</td>
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<td>0.8080</td>
<td>0.7931</td>
<td>0.7949</td>
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<td>Generator Reactive Outputs</td>
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<tr>
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<td>−3.00</td>
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Tab. 2: The first case-study (Scenario B): Simulation results obtained for BSA, ABC, GA, NRM and GWO algorithms.

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<th>GA</th>
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<td>$q_{g2}$</td>
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<td>0.0583</td>
<td>0.1882</td>
<td>0.1420</td>
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<tr>
<td>$v_1$</td>
<td>0.90</td>
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<td>1.0994</td>
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<tr>
<td>$v_2$</td>
<td>0.90</td>
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<td>1.0949</td>
<td>1.1000</td>
<td>1.0967</td>
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<tr>
<td>$v_3$</td>
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<td>1.0667</td>
<td>1.0697</td>
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<td>$v_4$</td>
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<td>1.0775</td>
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<td>1.0795</td>
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<td>$v_5$</td>
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<td>1.0725</td>
<td>1.0779</td>
<td>1.0744</td>
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<td>0.225</td>
<td>0.1913</td>
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<td>$q_{dc}$</td>
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<td>$q_{dr}$</td>
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<td>$q_{dl}$</td>
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<td>0.15</td>
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<td>20</td>
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<td>$\gamma$ (°)</td>
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and 8, of 315 MW active loads and 115 MVAR reactive loads. It is extended with a two-terminal HVDC link which replaces the AC line between buses 4 and 5 in the original WSCC 9-bus test system.

The convergence characteristics of the GWO optimization algorithm for the second case-study are presented in Fig. 9.

Table 3 represents the simulation results obtained when applying BSA [14], GA [13], and GWO algorithms to the second case-study.

### 5.3. Comparative Study

Table 4 and Tab. 5 compare the total cost and CPU time for all of the tested techniques in the first and second case-study, respectively.

When using various optimization algorithms including GWO, the comparative costs and CPU times of the two case-studies are represented in Fig. 10 and Fig. 11, respectively.

As shown in Fig. 10, the cost encountered by applying the proposed GWO algorithm is lower than that obtained when applying the BSA, ABC, GA, or NRM algorithm for the first case-study. This observation is still valid for the second case-study when comparing...
the cost of applying the GWO algorithm with that of applying BSA or GA.

According to Fig. 11, applying the proposed GWO algorithm attracts the lowest CPU time when compared with other optimization algorithms for the two cases studied in this paper.

6. Conclusion

In this paper, the OPF problem of a two-terminal HVDC transmission power system was addressed using the proposed GWO algorithm. The algorithm was applied to two HVDC test systems, namely the 5-bus and the WSCC 9-bus test systems.

The total costs and CPU times encountered when applying the GWO algorithm showed to be lower than those obtained when using other optimization algorithms, such as BSA, ABC, GA, and NRM, with a faster rate of convergence.

The GWO algorithm demonstrated several advantages such as fast convergence, adaptability and reliability to the optimal solution with a performance that was not sensitive to the initial conditions. Therefore, the scope of the future work is to apply the GWO algorithm to solve the OPF problems of large power systems equipped with FACTS devices and renewable energy sources.

References


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