Linear Model for Optimal Distributed Generation Size Prediction

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Abstract. This article presents a linear model predicting optimal size of Distributed Generation (DG) that addresses the minimum power loss. This method is based fundamentally on strong coupling between active power and voltage angle as well as between reactive power and voltage magnitudes. This paper proposes simplified method to calculate the total power losses in electrical grid for different distributed generation sizes and locations. The method has been implemented and tested on several IEEE bus test systems. The results show that the proposed method is capable of predicting approximate optimal size of DG when compared with precision calculations. The method that linearizes a complex model showed a good result, which can actually reduce processing time required. The acceptable accuracy with less time and memory required can help the grid operator to assess power system integrated within large-scale distribution generation.

Keywords
Distributed generation, linear load flow, power flow analysis, power losses, smart grid.

1. Introduction

Recent efforts towards rebuilding the electrical grid pose many challenges. The installation of renewable distributed generation as energy sources in the distribution system provides numerous benefits. Renewable DGs are the only option to a sustainable energy supply infrastructure since they are neither exhaustible nor polluting [1]. Moreover, DG has many advantages: it can relieve the congestions in the electrical network, it can defer transmission expansions and upgrades, it provides peak reduction of energy losses, it improves power quality and reliability, and it increases efficiency.

There is a vast range of terminologies used for “distributed generation,” such as “embedded generation” “dispersed generation,” or “decentralized generation,” [2]. DG can be defined as electric power source with small capacity connected directly to the distribution power network. In general, there are multiple types of DGs depending on the source of energy they use to produce electric power. The DG technologies include fuel cells, storage devices, and a number of renewable energy-based technologies; for example wind, photovoltaic, thermal, biomass, ocean, etc.

The most difficult part for most utilities (electric companies) is to determine site and sizing of the DG to be installed. The active power losses would increase because of the improper size of DG in comparison with systems without DG installed. This is the case even if the location is fixed, due to some other reasons in the system. Optimal placement and sizing depend on the type of DG as well. A critical review of different methodologies is needed to solve this problem. An overview of the state of the art, classifying and analyzing current and future research trends in the field of models and methods applied to the Optimal DG Problem (ODGP) has been demonstrated in [3]. Author shows an extensive number of articles that have been published on this topic, which indicates the existing interest in the subject of this research. Examples of recent publications are Refs. [4], [5], [6], [7], [8], [9] and [10].

In [4], there is a proposed structure, which can be considered as an efficient tool in planning and energy management of micro grids via optimizing the capacity and type of Distributed Generation (DG) sources. The optimal power flow and analytical method were used by authors of [5] to find the optimal size and site of distributed generators in radial systems. Different types
of DGs are considered, and their power factors are optimally calculated. The proposed method is faster and more accurate than previous analytical methods but it is the only suitable for radial systems.

Power system planning has many constraints, which can be considered as a multi-objective optimization problem. Determining the optimal sizes, sites of multiple DG and their generated power contract price was presented in [9]. This paper proposed multi-objective particle swarm optimization to improve voltage profile and stability, power-loss reduction, and reliability enhancement.

In radial distribution network, finding optimal DG locations and sizes by Teaching Learning Based Optimization (TLBO) technique was presented in [7]. An analytical approach has been developed to improve the network voltage profile in distribution systems by installing the most suitable size of DG at a suitable location. Authors tried to find a quick selection of DG parameters based on uniformly distributed loads represented by algebraic equations to achieve required levels of network voltage in a radial network [5].

Microgrid (MG) with different types of DG model is transformed to be a two-stage optimization problem to decide optimal sizes, locations and mix of dispatchable and intermittent Distributed Generators (DGs). The planning methodology includes long-term costs as investment, Operation and Maintenance (O&M), fuel and emission costs of DGs, while the income includes payment by MG loads and utility grid [9]. A Column and Constraint Generation (CCG) framework is compared with conventional MG planning approaches to demonstrate the effectiveness of the proposed methodology. Another mixed integer nonlinear model was used to find the optimal DG site and size problem [10]. Modified Teaching-Learning Based Optimization (MTLBO) algorithm is proposed to minimize total power losses, which is considered as objective function. The proposed algorithm was compared with the performance of brute force method.

An approach was proposed in [11] to find out the optimal size and power factor of the four types of dispatchable DGs. In [12], the effect of renewable DGs allocation on minimizing energy losses was demonstrated through a multi-period AC optimal power flow based technique, taking into consideration smart control schemes. The voltage profile and significant reduction in network power losses were the main objective function in that work.

The network investment deferral incentives and active loss reduction have been investigated as the objective function to maximize the advantages of Distribution Network Operators (DNOs) [13]. The optimal location and size of DGs in the UK are found considering current Ofgem financial incentives for DNOs.

The typical Optimal DG Placement (ODGP) problem deals with the determination of the optimal locations and sizes of multi DGs to be installed into existing distribution networks, subject to electrical network operation constraints and DG, and investment constraints. The ODGP is solved by a heuristic iterative strategy in two stages, in which clustering technique method and exhaustive search are exploited [14]. The optimal DG locations are computed based on increasing of bus voltage sensitivities and the optimal size of DGs which is calculated by Differential Evolution (DE) [15].

Previous studies have primarily concentrated on solving the problem of DG placement and size into electrical network. In summary, important researches could be classified into three different categories: planning, re-design and operation. In operation methodology, most of researches tried to reduce time of calculations by DC power flow. Hence, in this paper, an attempt is made to develop simple, efficient, and flexible linear expression between voltage angles and real power losses. By using this expression, a solution of optimal DG sizing can be predicted and calculated quickly and easily in distribution network. Furthermore, we will concentrate on DGs connected to Middle Voltage (MV) distribution grid, which is mostly for wind power. The proposed method can help utility operators to select the optimal size of DGs.

2. \( \mathbf{P} \delta - \mathbf{QV} \) Decoupling

The AC power flow problem has been formulated at Newton-Raphson method by the following power flow equations:

\[
J(x) \left( \frac{\Delta P(x)}{\Delta V} \right) + \left( \frac{\Delta Q(x)}{\Delta V} \right) = 0. \tag{1}
\]

The function \( \Delta P(x) \) is active mismatches and \( \Delta Q(x) \) is reactive power mismatches. The solutions are determined approximately by considering first term of Taylor series:

\[
J(x) \left( \frac{\Delta \delta}{\Delta V} \right) = 0. \tag{2}
\]

The partial derivative values of the Jacobian matrix, which are either \( P \) or \( Q \) with respect to either \( |V| \) or \( \delta \), are defined as:

\[
J(x) = \begin{bmatrix}
\frac{\partial P}{\partial \delta} & \frac{\partial P}{\partial |V|} \\
\frac{\partial Q}{\partial \delta} & \frac{\partial Q}{\partial |V|}
\end{bmatrix}. \tag{3}
\]
The matrices $\frac{\partial P}{\partial \delta}$, $\frac{\partial Q}{\partial |V|}$ and $J$ in Eq. (3) are always quadratic. The four matrix variables associated with each power network node $k$:

- $P_k$ the net active power (generation – load) at node $k$.
- $Q_k$ the net reactive power (generation – load) at node $k$.
- $V_k$ the voltage magnitude at node $k$.
- $\delta_k$ the voltage magnitude at node $k$.

For transmission line systems, the strong coupling between $P$ and $\delta$, as well as between $Q$ and $V$ can be observed normally. This property will be used in this section to accelerate and simplify the computations. Then, it will derive a linear approximation called linearization of power flow in the next section. This model will have linear relation between the bus active power $P$ and voltage angle $\delta$.

Let us consider a simple model of a transmission line system (k to m) as in Fig. 1. The active and reactive power flows can be expressed mathematically as follows:

\[
P_{km} = V_k^2 G_{km} - V_k V_m G_{km} \cos \delta_{km} - V_k V_m B_{km} \sin \delta_{km},
\]

\[
Q_{km} = -V_k^2 (B_{km}) + V_k V_m B_{km} \cos \delta_{km} - V_k V_m G_{km} \sin \delta_{km},
\]

where $G_{km}$ is the transmission line conductance ($1/R_{km}$), $B_{km}$ is the transmission line susceptance ($1/X_{km}$), $X_{km}$ is the series reactance of the line and $R_{km}$ is the series resistance of the line.

\[\text{Fig. 1: Power flow in transmission line.}\]

If the series resistance and the shunt admittance are both neglected and assumed to be zero, then above expressions can be simplified as:

\[
P_{km} = \frac{V_k V_m \sin \delta_{km}}{x_{km}},
\]

\[
Q_{km} = \frac{V_k^2 - V_k V_m \cos \delta_{km}}{x_{km}}.
\]

As shown in Fig. 2, the voltage angles are relatively small in the usual range of operations; a strong coupling between active power and voltage angle as well as between reactive power and voltage magnitudes exists. Whereas much weaker coupling between active power and voltage magnitude, and between reactive power and voltage angle exists. When $\delta_{km}$ is near to 90°, strong coupling between $P$ and $V$ as well as between $Q$ and $\delta$ is observed clearly [16].

\[\text{Fig. 2: P-\delta and Q-\delta relation for a transmission line.}\]
With this assumption, the elements of Jacobian Matrix can be considered as:

\[
\mathbf{J}_{DEC} = \begin{bmatrix}
\frac{\partial P}{\partial \delta} & 0 \\
0 & \frac{\partial Q}{\partial |V|}
\end{bmatrix}.
\] (16)

Thus, the updates of voltage magnitudes and angles have no coupling between them, and Eq. (2) can be divided into two uncoupled equations:

\[
\frac{\partial P}{\partial \delta}(\Delta \delta) + \Delta P(\Delta \delta, V) = 0,
\] (17)

\[
\frac{\partial Q}{\partial \delta}(\Delta V) + \Delta Q(\Delta V) = 0.
\] (18)

The solution can be even faster (but then only an approximation) if approximations concerning the active and reactive power mismatches are used.

3. Power Flow Linearization Model

In this section, linearization equations of power flow will be derived. Again, considering transmission line in section II with expressions for the active power flows \(P_{km}\) and \(P_{mk}\) can be represented by [17]

\[
P_{km} = V_k^2 G_{km} - V_k V_m G_{km} \cos \delta_{km} - V_k V_m B_{km} \sin \delta_{km},
\] (19)

\[
P_{mk} = -V_m^2 G_{km} + V_k V_m G_{km} \cos \delta_{km} - V_k V_m B_{km} \sin \delta_{km}.
\] (20)

The active power losses can be determined by using the transmission line equations (Eq. (19) and Eq. (20)) to get:

\[
P_{km} + P_{mk} = G_{km}(V_k^2 + V_m^2 - 2V_k V_m \cos \delta_{km}).
\] (21)

If the terms corresponding to the active power losses in Eq. (19) and Eq. (20) are ignored, the result is:

\[
P_{km} = -P_{mk} = -V_k V_m B_{km} \sin \delta_{km}.
\] (22)

During light load conditions, the following additional approximations are particularly valid:

\[
V_k \approx V_m \approx 1p \cdot u,
\] (23)

\[
\sin \delta_{km} \approx \delta_{km}.
\] (24)

And since

\[
B_{km} = -\frac{1}{x_{km}},
\] (25)

the expression of active power flow \(P_{km}\) can be simplified to:

\[
P_{km} = \frac{\delta_{km}}{x_{km}} - \frac{\delta_k - \delta_m}{x_{km}},
\] (26)

\[
P_{km} = x_{km}^{-1} \delta_{km},
\] (27)

where \(P_{km}\) is the power flow in transmission line \((k\text{ to } m)\), \(\delta_k\) and \(\delta_m\) are the voltages angles at terminals of line and \(x_{km}\) is the reactance of transmission line \((k\text{ to } m)\).

Thus, for \(k = 1, 2, \ldots, N\), where \(N\) is the number of buses in the electrical network. The active power injection at bus \(k\) is calculated by

\[
P_k = \sum_{m\in N} x_{km}^{-1} \delta_{km} = \left(\sum_{m\in N} x_{km}^{-1}\right) \delta_k + \sum_{m\in N} (-x_{km}^{-1} \delta_{km}).
\] (28)

This can be written into matrix form as follows:

\[
\mathbf{P} = \mathbf{B} \Delta \delta,
\] (29)

where \(\mathbf{P}\) is the vector of the net power injections at node \(k\) and \(\mathbf{B}\) is the nodal admittance transposed matrix with the following elements:

\[
B_{km} = x_{km}^{-1}, \quad B_{kk} = \sum_{m\in N} x_{km}^{-1}.
\] (30)

The determinant of matrix \(\mathbf{B}\) in Eq. (29) is equal to zero, singular matrix. Where slack node has zero angle reference, i.e. \(\delta_{ref} = 0\), that means one of the equations in the system is removed. Of course, the angle reference is still needed but the bus associated with that row and column will be disposed. This means that the equations in Eq. (29) have no unique solution and that the rows of matrix \(\mathbf{B}\) are linearly dependent.

4. Simplified Power Losses Method

Transmission loss allocation is not a simple task even in a simple two-node system with a single load supplied by one generator. In a real system, matters get more complicated because of two facts. The first is that the determination of the line power flows caused by each load through each transmission line has a good degree of arbitrariness. The second is that the loss of transmission line is a nonlinear function of the line flow.
Losses are distributed across all buses, according to their level of generation or consumption only. The slack bus is just a phase reference bus and like other generators, bus is not in charge for compensating the total loss of system but also divide the losses between generators and loads. Therefore, power system analysis will be performed to calculate exact total losses.

Some researches [18] and [19] used and implemented simple method for two loads in different locations but these loads were not identical with two real demands. These two demands were located far away from generators and near generators respectively, to be allocated with the same amount of losses. However, the network topology was never taken into account.

The method to calculate active power losses can be presented as simplified equation, at any line, losses onto the series impedance of a transmission line are $I^2 \cdot R$ and $I^2 \cdot X$:

$$
I = \left( \frac{P - JQ}{V^*} \right), \quad (31)
$$

$$
I^* = \left( \frac{P + JQ}{V} \right), \quad (32)
$$

$$
I^2 = I \cdot I^* \left( \frac{(P - JQ)(P + JQ)}{V \cdot V^*} \right) = \frac{(P^2 + Q^2)}{V^2}, \quad (33)
$$

$$
P_{\text{losses}} = I^2 \cdot R = \left( \frac{P^2 + Q^2}{V^2} \right) \cdot R, \quad (34)
$$

$$
Q_{\text{losses}} = I^2 \cdot X = \left( \frac{P^2 + Q^2}{V^2} \right) \cdot X. \quad (35)
$$

That means, transmission loss is dividing into two components. The first is due to active power loss and second is reactive power loss. In this paper, interest will be focused on active or real power losses which are more affected by DG. Most of DG (active power generation) is located near loads to reduce total losses. The difference between the voltages of generators will reduce the losses due to the reduction of current flow from generators to loads. So that, reactive power flow will be the same for both nonlinear and linear model and can be neglected.

The DG also improves voltage profile in system but power flows will be slightly different. In addition, voltage values at each bus are around 1 p.u, which can simplified by the equation of active power flow as follows:

$$
P_{\text{losses}} = P^2 \cdot R. \quad (36)
$$

So that, direct relation between voltage angles and power losses can be expressed by:

$$
P_{\text{losses}} = B^2 \cdot \Delta \delta^2 \cdot R. \quad (37)
$$

5. Procedure and Results

The problem for prediction of the optimal sizes of DG at each load bus in different IEEE standard systems has been formulated as linear state equations. However, these predictions are not the optimal sizes of the DGs, because in addition to the fact that they ignore the reactive power flow in lines and all voltage values are assumed to be $1p \cdot u$, an approximation method was used to calculate active power flows. The procedure presented in this paper has smooth properties, which lead to the solution that is close to optimal values with less iterations, less consumed memory and shorter computation times. In addition, the error from linearization method can be reduced during the estimation process.

Firstly, the systems with (5, 14, 30) buses have been tested with Newton Raphson load flow method to calculate power flows at each bus. Then, the exact total power losses without DG have been found. These calculations have been repeated with added DG inside these systems at different locations in load buses. At each location, the DG output has been initialized with 1 MW at beginning then it increased by 1 MW until it has reached the maximum value (total load demand) required for the whole system. This is illustrated in Fig. 3, Fig. 4 and Fig. 5, each of three curves indicate the total power loss (for each system) which has varied according to the size of DG in different locations.

The figures show that at each bus, losses have similar behavior that can be divided into two parts. In first part, the losses curves are beginning to be close to linear part. When power output (MW) of DG is increased, losses decreased until they reach the minimum value. Then, it is nonlinear in the second part; when power output (MW) of DG continues increasing while losses also will increase in the grid.

![changing of power losses with DG](image)
In linear part, curves of losses will continue decreasing in parallel but sometimes they intersect each other at non-linear part. Most of researchers have been considered the lower losses curve as optimal location. In fact, there is an optimal size of DG at each location for the total losses but it is not necessary to be at the lower one. The principle of using linear model is to find the estimated optimal size of DG based on the linear part of these curves. In addition, the linear relations between the changing of voltage magnitude and angles, and increasing of power generation of DG are illustrated in Fig. 6, Fig. 7 and Fig. 8.

The procedure begins to compute total active power losses in lines before DG, which will be located further in different load buses. DG at each load bus has been initialized with small amount (1 MW). The power flow computation based on the linear method proposed and initial power loss is obtained by simplified active power losses calculation. Then, the information on the individual power loss corresponding to DG size increased by 1 MW are stored in $P_{losses}$ (n) and DG (i, n) respectively. The minimum value of these losses are selected by comparing the sizes of DG’s in stored memory when DG reached the maximum total demand of the system.

Actual values for DG and total power loss at each location are required to verify whether the optimal sizes of DG estimated by the linear model proposed are acceptable. These values of power losses are summarized by the following steps:

- Find initial voltage angles and active power losses by Newton Raphson method,
- build impedance matrix (X) for system,
- choose location of DG at one of load buses,
- increase DG power 1 MW at each step,
- calculate active power flow from Eq. (27),
- find summation of power losses in system by Eq. (37).
• compare losses for last two cases and save minimum results,
• repeat steps from 4 till generation of DG reach total demand,
• determine minimum losses and size of DG,
• repeat steps from 3 for all load system buses.

Lately, the accumulated data of the minimum power loss, location and size of DG are obtained. The results of proposed Linear Method (LM) have been validated with exact solution by using Newton Raphson method (NR) for (14 and 30) bus as shown in Fig. 9 and Fig. 10.

It is observed that each state converges sufficiently for different systems and it can predict optimal size of DG for minimum losses. Table 1 shows the significant execution times saving by proposed method (LM) compared to that required by NR. Time complexity analysis of LM and NR is performed on a computer with 2.7 GHz CPU, core i7-4800MQ, RAM of 16 GB and Windows 7 operating system running in real-time mode. The reduction of time and memory consumption required by proposed linear model will help control center of utilities by providing real-time data. In addition to helping utilities improve efficiency, linear model will be designed to speed up integration of DG into grids.

Tab. 1: Comparison of LM and NR time consumptions.

<table>
<thead>
<tr>
<th>System</th>
<th>Time Consumptions (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>NR Method</td>
</tr>
<tr>
<td>Bus 5</td>
<td>0.2888</td>
</tr>
<tr>
<td>Bus 14</td>
<td>4.17</td>
</tr>
<tr>
<td>Bus 30</td>
<td>28.95</td>
</tr>
</tbody>
</table>
6. Conclusion

Linear equations modelling and simplified power losses calculation have been used to predict approximate optimal DG size. Each location of DG at load buses has optimal size of DG for minimum power losses. It is noted that optimal size for optimal location is not necessary to be the same as optimal location for optimal size. The proposed method has acceptable accuracy with less time and memory consumption where it is crucial factor in real-time management of power grids. The loss reduction by properly placed and appropriate size of DG is one of the more significant findings to emerge from this study. With these benefits, control and assessment of large scales power grid will become easily predictable as more intermittent power sources, such as wind and solar, come online. Different IEEE test bus systems have been tested and results are validated with exact calculations.

References


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