SYSTEM FAILURE RATE CALCULATION

N. Hanusová, J. Zahradník

AŽD Praha s.r.o., Závod Technika, Výskum a vývoj, Bytčická 82, 010 01 Žilina
Katedra riadiacich a informačných systémov, Elektrotechnická fakulta, Žilinská univerzita, Veľký diel, 010 26 Žilina

Summary The article deals with methodology of a system failure rate calculation that is based on the adoption of the condition $\lambda.t \ll 1$. Assuming that the condition is satisfied, the serial-parallel system failure rate calculation is easier and the results are either the same or worse from a view of system reliability evaluation. So the results can be accepted. A simple example will be given as a confirmation of the assertion.

1. INTRODUCTION

From the view of a particular elements failure rate impact to system reliability it is possible to divide systems into three base types: serial, parallel and serial-parallel systems. Consider i-th system element, the probability of its failure-free state occurrence is R_i and the probability of its failure state occurrence is F_i. Similarly the probabilities for a whole system are R_S and F_S.

2. SERIAL SYSTEM

Object reliability is represented by the serial system from Fig. 1 if a failure of any component of the object results in a failure of the object as a whole without regard to structural and technological object construction.

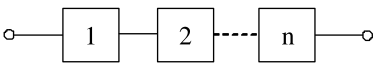


Fig. 1. The serial system block diagram.

The serial system is in failure-free state if all its elements are in failure-free state in a given moment. In case of mutual independence of individual components failure occurrences and assuming that the distribution of time to occurrence of an element failure is exponential *(1)*, the serial system, composed of n elements, is described by equations *(2)* and *(3)*.

$$R_i(t) = e^{-\lambda_i.t} \tag{1}$$

$$R_S(t) = \prod_{i=1}^{n} R_i(t) = \prod_{i=1}^{n} e^{-\lambda_i.t} = e^{-\sum_{i=1}^{n}\lambda_i.t} = e^{-\lambda_S.t} \tag{2}$$

$$F_S(t) = 1 - R_S(t) \tag{3}$$

The failure rate relation *(4)* follows from the equation *(2)*.

$$\lambda_S = \sum_{i=1}^{n} \lambda_i \tag{4}$$

It follows that the probability distribution of serial system failure occurrence is exponential if all elements failure distributions are exponential too. The total failure rate is a sum of all individual elements failure rates.

3. PARALLEL SYSTEM

Object reliability is represented by the parallel system from Fig. 2 if a failure of all components of the object results in a failure of the object as a whole without regard to structural and technological object construction.

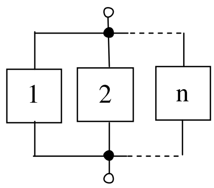


Fig. 2. The parallel system block diagram.

The parallel system is in failure-free state if at least one its component is in failure-free state. The parallel system, composed of n components, is described by equations (5) and *(3)*.

$$R_S(t) = 1 - \prod_{i=1}^{n}\left(1 - R_i(t)\right) = 1 - \prod_{i=1}^{n}\left(1 - e^{-\lambda_i.t}\right) \tag{5}$$

The dependence of time in equation *(5)* is not the exponential distribution. Assuming that $\lambda.t \ll 1$ *[1]* what is mostly satisfied by using of electronic components nowadays, the probability of element failure-free state occurrence may be expressed by the equation *(6)*.

$$R_i(t) = e^{-\lambda_i.t} \cong 1 - \lambda_i.t = \tilde{R}_i(t) \tag{6}$$

At first the probability of element reliable operation is calculated by the equation *(1)*. The corresponding values are represented in graph in Fig. 3 as $R_1(t)$ for λ_1, $R_2(t)$ for λ_2, $R_3(t)$ for λ_3. Next the probability of element reliable operation is calculated by the equation *(6)* for the exponential distribution approximation. The corresponding values are represented in graph in Fig. 3 as $\tilde{R}_1(t)$ for λ_1, $\tilde{R}_2(t)$ for λ_2, $\tilde{R}_3(t)$ for λ_3.

Considered initial values are:

$\lambda_1 = 1.10^{-7}$ h^{-1}, $\lambda_2 = 1.10^{-6}$ h^{-1}, $\lambda_3 = 1.10^{-5}$ h^{-1} and the time range is t = <0 - 200000> h.

The dependences of both probabilities $R_i(t) = e^{-\lambda_i.t}$ and $\tilde{R}_i(t) = 1 - \lambda_i.t$ on the $\lambda_i.t$ are represented in graphs in Fig. 4.

The results obtained from equations *(1)* and *(6)* are equal at most, so the evaluation of element reliability by the approximate equation is worse than by equation *(1)*, therefore it can be accepted.

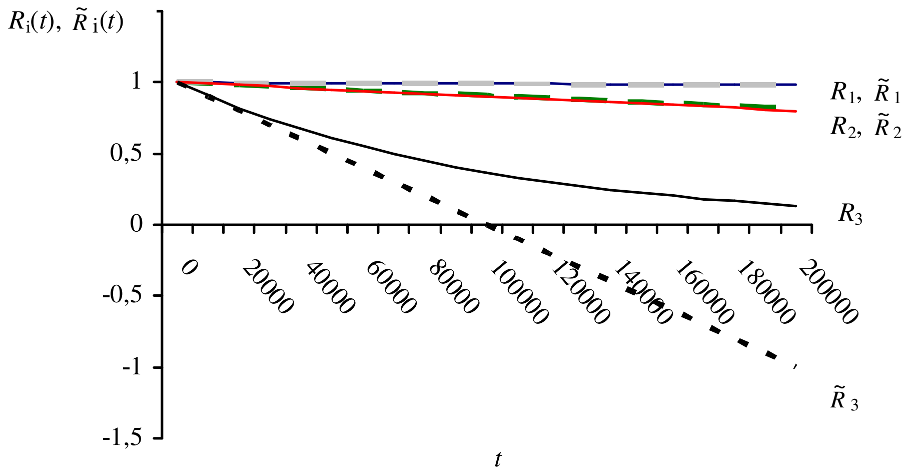


Fig. 3. The comparison between element reliable operation probabilities values calculated by equations for exponential distribution and exponential distribution approximation.

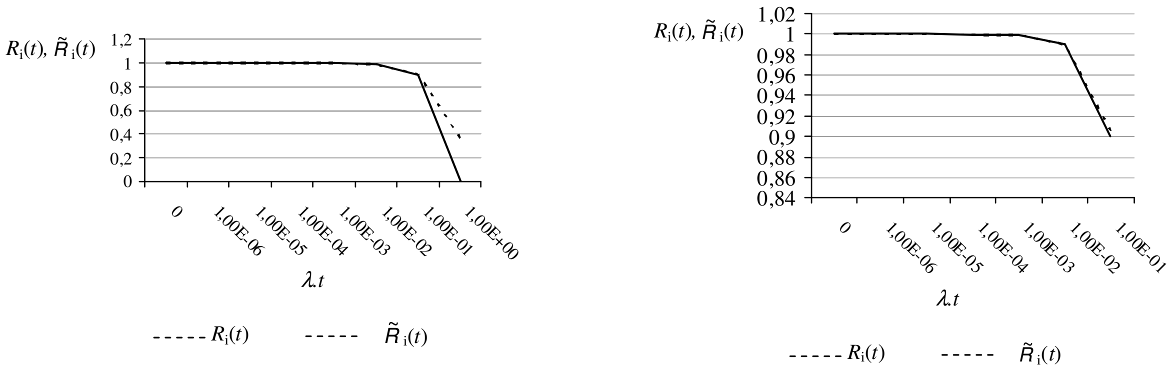


Fig. 4. The representation of component reliable operation results obtained from differenced calculations.

Assuming that the distribution of failure occurrence probability is exponential, the element failure rate is counted by equation *(7)*.

$$F(t) = 1 - e^{-\lambda.t} \Rightarrow \lambda(t) = \frac{\frac{dF(t)}{dt}}{1 - F(t)} \tag{7}$$

The element failure rate for the approximate exponential distribution may be calculated by the equation *(8)*.

$$\tilde{F}_i(t) = \lambda_i.t \Rightarrow \tilde{\lambda}_i(t) = \frac{d\tilde{F}_i(t)}{dt} \tag{8}$$

The equations for approximate calculation of both failure-free state and failure state occurrence probabilities may be rewritten to (9) and (10).

$$\tilde{R}_S(t) = 1 - \prod_{i=1}^{n}\left(1 - \left(1 - \lambda_i.t\right)\right) = 1 - \prod_{i=1}^{n}\lambda_i.t \tag{9}$$

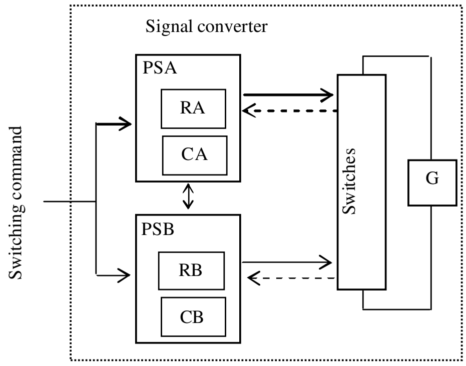
$$\tilde{F}_S(t) = 1 - \tilde{R}_S(t) = \prod_{i=1}^{n}\left(1 - \left(1 - \lambda_i.t\right)\right) = \prod_{i=1}^{n}\lambda_i.t \tag{10}$$

The failure rate of the parallel system then may be calculated by the equation *(11)*.

$$\frac{d\tilde{F}_S(t)}{dt} = n.\left(\prod_{i=1}^{n}\lambda_i\right).t^{n-1} \tag{11}$$

4. THE FAILURE RATE CALCULATION EXAMPLE

The chapter shows the example of the simple object represented by the block diagram in Fig. 5. It is a part of a signal converter for green railway signal control provided that switches are failure free and units CU and C in each processing station PS are independent. The system failure occurrence is represented by the top event using FTA method in Fig. 6.



PSA, PSB processing station A, processing station B
CA, CB comparator A, comparator B
CUA, CUB computation unit A, computation unit B
G green signal lamp

Fig. 5. The block diagram of the signal converter for green railway signal control.

Minimal cut sets of the tree from Fig. 6 are *(12)*.

(12)

It is clear that the object behavior can be described

$O = CA.CB + PSA.PSB$ *by a serial-parallel system. The minimal cut sets are statistically independent, so a probability of the top event (O) occurrence can be calculated by the equation*

(13).

$$P(O) = F_S = 1-\left(1-F_{CA}.F_{CB}\right).\left(1-F_{CUA}.F_{CUB}\right) = \\ = F_{CUA}.F_{CUB} + F_{CA}.F_{CB} - F_{CUA}.F_{CUB}.F_{CA}.F_{CB} \tag{13}$$

Following values are used:
$t = 10^5$ h , $\lambda_{CUA} = \lambda_{CUB} = \lambda_{CA} = \lambda_{CB} = 2.10^{-8}$ h^{-1}.
Corresponding failure occurrence probabilities are:
$F_{CUA} = F_{CUB} = F_{CA} = F_{CB} = 0{,}001998$.

By insertion the values to the equation

(13) the final probability of the system failure occurrence is:
$F_S = 7{,}98399.10^{-6}$

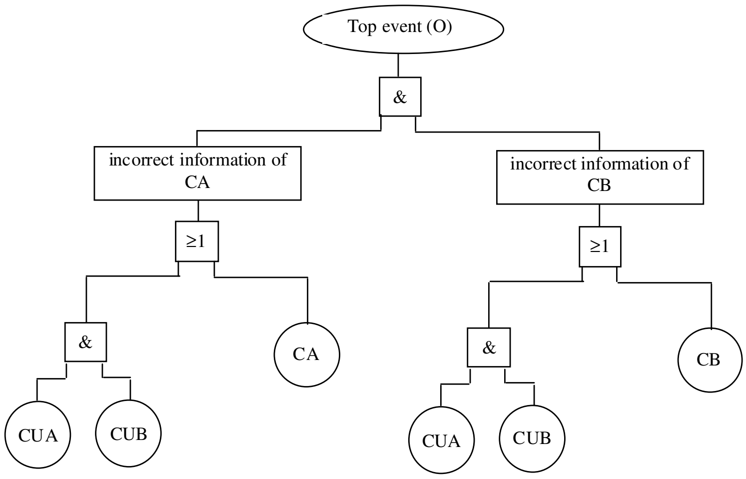


Fig. 6. The failure state tree for system failure state occurrence analysis.

By rewriting the equation

(13) to dependence of time we obtain equation *(14)*.

$$F_S(t) = \left(1-e^{-\lambda_{CUA}.t}\right).\left(1-e^{-\lambda_{CUB}.t}\right)+ \\ +\left(1-e^{-\lambda_{CA}.t}\right).\left(1-e^{-\lambda_{CB}.t}\right)- \\ -\left(1-e^{-\lambda_{CUA}.t}\right).\left(1-e^{-\lambda_{CUB}.t}\right).\left(1-e^{-\lambda_{CA}.t}\right).\left(1-e^{-\lambda_{CB}.t}\right) \tag{14}$$

By derivation of function $F_S(t)$ by the time we obtain an equation *(15)*.

$$\frac{dF_S(t)}{dt} = \left(\lambda_{CUA}+\lambda_{CA}\right)e^{-(\lambda_{CUA}+\lambda_{CA}).t}+ \\ +\left(\lambda_{CUA}+\lambda_{CB}\right)e^{-(\lambda_{CUA}+\lambda_{CB}).t}+ \\ +\left(\lambda_{CUB}+\lambda_{CA}\right)e^{-(\lambda_{CUB}+\lambda_{CA}).t}+ \\ +\left(\lambda_{CUB}+\lambda_{CB}\right)e^{-(\lambda_{CUB}+\lambda_{CB}).t}- \\ -\left(\lambda_{CUA}+\lambda_{CA}+\lambda_{CB}\right)e^{-(\lambda_{CUA}+\lambda_{CA}+\lambda_{CB}).t}- \\ -\left(\lambda_{CUB}+\lambda_{CA}+\lambda_{CB}\right)e^{-(\lambda_{CUB}+\lambda_{CA}+\lambda_{CB}).t}- \\ -\left(\lambda_{CUA}+\lambda_{CUB}+\lambda_{CA}\right)e^{-(\lambda_{CUA}+\lambda_{CUB}+\lambda_{CA}).t}- \\ -\left(\lambda_{CUA}+\lambda_{CUB}+\lambda_{CB}\right)e^{-(\lambda_{CUA}+\lambda_{CUB}+\lambda_{CB}).t}+ \\ +\left(\lambda_{CUA}+\lambda_{CUB}+\lambda_{CA}+\lambda_{CB}\right)e^{-(\lambda_{CUA}+\lambda_{CUB}+\lambda_{CA}+\lambda_{CB}).t}$$

(15)

$$\lambda_S(t) = \frac{\frac{dF_S(t)}{dt}}{R_S(t)}$$

System failure rate may be calculated by the equation *(16)* as mentioned above.

(16)

Assuming that condition $\lambda.t \ll 1$ is true, the system failure state probability may be expressed by the equation *(17)*.

$$\tilde{F}_S = F_{CUA}.F_{CUB} + F_{CA}.F_{CB} - F_{CUA}.F_{CUB}.F_{CA}.F_{CB} = \lambda_{CUA}.\lambda_{CUB}.t^2 + \lambda_{CA}.\lambda_{CB}.t^2 - \lambda_{CUA}.\lambda_{CUB}.\lambda_{CA}.\lambda_{CB}.t^4 \qquad (17)$$

And system failure rate based on the analyzed serial-parallel system is expressed by the (18).

$$F_S = F_{CUA}.F_{CUB} \quad or \quad F_{CA}.F_{CB} \Rightarrow \qquad (18)$$

$$\Rightarrow \tilde{\lambda}_S = \lambda_{CUA}.\lambda_{CUB}.2.t + \lambda_{CA}.\lambda_{CB}.2.t$$

Results for the analyzed system are represented in the graph in Fig. 7. The graph represents calculated results of:

- Probabilities of system failure occurrence (calculated by different approaches) $F_S(t)$, $\tilde{F}_S(t)$.
- System failure rates (calculated by different approaches) $\lambda_S(t)$, $\tilde{\lambda}_S(t)$.
- System failure rate that is not allowed to be exceeded (e.g. maximum acceptable level for the analyzed system/function) $\lambda_{SMAX}(t)$.

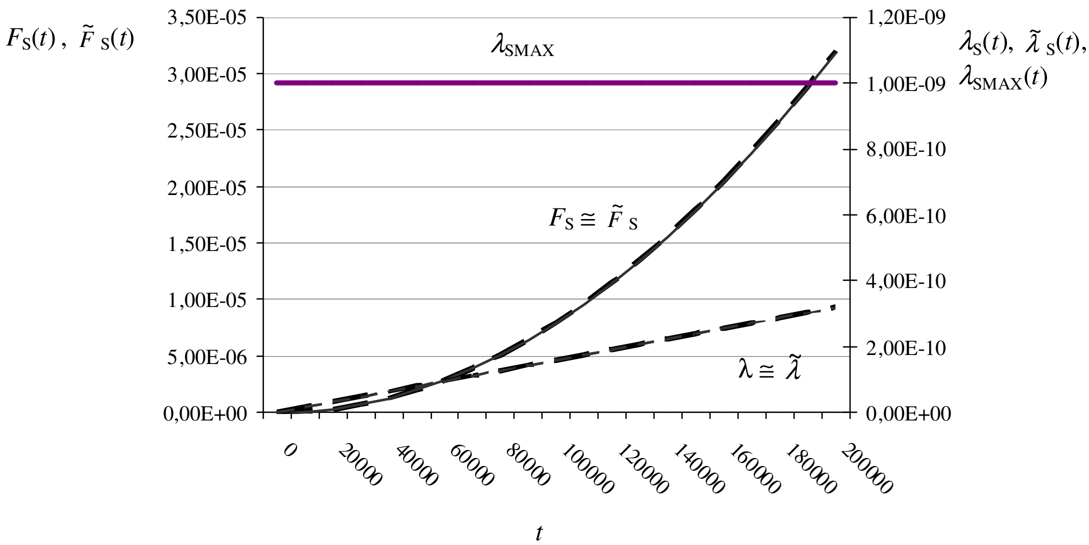


Fig. 7. The comparison of "classical" and approximated calculation results.

5. CONCLUSION

The article presents results of two approaches to the system failure rate calculation for serial – parallel systems. While the considered condition $\lambda.t \ll 1$ is true, both approaches are equivalent for system failure rate calculation. The approximated system failure rate equation is simpler for quantification. So the described approach may be used for the most electronic systems used at present.

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