DIFFERENT VIEW ON PQ THEORY USED IN THE CONTROL ALGORITHM OF ACTIVE POWER FILTERS

R. Pavlanič, M. Marinelli, B. Zigmund

1University of Zilina in Zilina, Faculty of Electrical Engineering, Department of Mechatronics and Electronics, Univerzitná 8, 010 26 Zilina, Slovakia, pavlanič@fel.utc.sk, brano.zigmund@gmail.com,
2Politecnico di Bari, Dipartimento di Elettrotecnica ed Elettronica, V. Orabona 4, 70125 Bari, Italy, marinelli@deemail.poliba.it

Summary

The improvement of power quality is a frequently discussed issue, which still requires a considerable research effort to be devoted to the study of the problem. The aim of this paper is to describe some problems related to the control of switching compensators, commonly known as active power filters. It also includes some shortcomings of pq theory regarded as three phase instantaneous power theory. The term “shortcomings” means that the pq theory does not provide a proper description of power properties. Moreover, the control algorithm based on this theory only achieves satisfactory results for sinusoidal balanced voltage system. Nevertheless, it can still be considered a helpful approach to the problem under study. The simulation results presented in this paper illustrate the weaknesses of the pq theory.

1. INTRODUCTION

Increasing number of power electrical and electronic equipment which affect negatively the AC main caused the question of power quality became very important. Reactive power, harmonics and also unbalance are coupled with low level of power quality. Nevertheless, a lot of equipment needs reactive power (electrical motors, transformers etc.) or harmonics (semiconductor converters, inductive furnaces, lighting and heating equipment etc.) in order to work properly. Consequences of the negative effects of such disturbances are increasing costs for delivering power (higher losses, lower efficiency), faster ageing process of equipment, unpredicted faults in power supply, communication interference and so on. Therefore it is desirable to compensate these negative effects as much as possible and also as closest as possible to the source of disturbances. In general it is possible to say that an ideal load has to draw from main only active power. This means all loads connect to the mains have to behave like a resistor (current has to follow the voltage waveform) and has to be in phase with voltage. These properties can be described by power factor given by:

\[
\Lambda = \frac{P}{S} \tag{1}
\]

where P is active power and S is apparent power. That means it is desirable to connect to the mains only loads with unity power factor. The apparent power is in this case defined as:

\[
S = \sqrt{P^2 + Q^2 + D^2} \tag{2}
\]

Where Q is reactive power and D is distortion power. Nevertheless, real loads need reactive and distortion (harmonics) power for their correct operation. It is necessary to compensate required reactive and distortion power through the use of dedicated devices. Nowadays many compensation techniques are presented. Passives techniques as capacitors battery for a compensation of reactive power, or passive serial filters tuned for compensation specific harmonic have a lot of disadvantage, for example: massiveness, unexpectedly resonances coupled with another device and so on. Diode rectifiers with boot chopper (also power factor corrected AC-DC converters) and PWM converters as line-side rectifiers can work with unity power factor. These converters are inseparable input part of inverters, so they can compensate only this device. Providing compensation of several devices together is desirable to use active power filter (APF). Considering a progress in development of semiconductor components is possible to use APF in higher power systems (also considering multilevel topologies etc.), but usually as medium power compensator. Shunt APF provides compensation of nonlinear load to unity power factor (but usually only displacement power factor and lower current harmonics of nonlinear load). Finally result is that the current drawn from main is sinusoidal.

2. SHUNT ACTIVE POWER FILTER TOPOLOGY

As fig.1 shows shunt active power filter is connected in parallel to nonlinear load and to main. Normally works as voltage source inverter VSI with current control loop.

\[
i_s(t) = i_r(t) - i_i(t) \tag{3}
\]

According to Fourier’s analysis distorted periodic function (non-harmonic current of load) can
be given by sum of sinusoidal functions (sum of harmonics), after load current is:

\[ i_L(t) = I_0 + \sum_{i=1}^{n} I_{m_i} \sin(i\alpha - \varphi_i) \]  
(4)

where \( I_0 \) is DC component of current (we will not consider it), \( I_{m_i} \) is amplitude of \( i \)th harmonic, \( i \) is harmonic order and \( \varphi_i \) is phase angle of \( i \)th harmonic. In general we can consider for line current the following equation:

\[ i_L(t) = I_{m_L} \sin(\alpha) \]  
(5)

where \( I_{m_L} \) is the amplitude of active part of current fundamental component. Current defined by (5) is in vector representation, first component of (6) presents reactive power, which is for four wire system given by:

\[ p(t) = u_a(t)i_a(t) + u_b(t)i_b(t) \]  
(11)

\[ q(t) = u_b(t)i_a(t) - u_a(t)i_b(t) \]  
(12)

(note: in some literature we can meet with different signs in (12), it is about standards of inductive and capacitive character of reactive power).

Using the inverse transformation \[ \psi \rightarrow \phi \] abc can be obtained expression of “instantaneous active and reactive power” defined by instantaneous phase voltages and currents as:

\[ p(t) = u_a(t)i_a(t) + u_b(t)i_b(t) + u_c(t)i_c(t) \]  
(13)

\[ q(t) = \frac{u_a(t) - u_b(t)}{\sqrt{3}} i_a(t) + \frac{u_b(t) - u_c(t)}{\sqrt{3}} i_b(t) + \frac{u_c(t) - u_a(t)}{\sqrt{3}} i_c(t) \]  
(14)

Expressions in parentheses present system of voltages which are phase shifted by 90°, \( u_a, u_b, u_c \) that is:

\[ q(t) = u_a(t)i_a(t) + u_b(t)i_b(t) + u_c(t)i_c(t) \]  
(15)

Where for voltages is valid:

\[ u_a(t) = u_a \left( t + \frac{T}{4} \right), \quad u_b(t) = u_b \left( t + \frac{T}{2} \right), \quad u_c(t) = u_c \left( t + \frac{3T}{4} \right) \]  
(16)

In vector representation they are voltage vectors perpendicular to the phase voltage vectors (as shown in fig.2).

**Fig. 2 Vector expression of \( u'_a, u'_b, u'_c \)**

**4. SEVERAL SHORTCOMINGS OF PQ THEORY**

In many publications which deal with the control algorithm for shunt APF, we can meet with interpretation of pq theory as theory about three phase instantaneous power (for example [3]). Call this theory by this terminology is not quite correct even in some case it can be misleading. Objective of this part is not to contradict quality of pq theory only point of some misinterpretations. The circuit shown in fig.3 will be used like an example to explain some inadequacies of pq theory. There is three phase power supply which contains of fundamental and 5th harmonic components (non-harmonic). This source supplies three phase non-linear load which draw non-harmonic current. Load current contains fundamental component and 5th harmonic components too. Let us assume that others voltage and current harmonics are negligibly small.

\[ p(t) = u_a(t)i_a(t) + u_b(t)i_b(t) \]  
(10)

On the base of instantaneous \( \phi \beta \) components Akagi et al defined “instantaneous active and reactive power” by following equations:
Different view on PQ theory used in the control algorithm...

In mathematical expression for voltages is valid following equations:

\[ u_a(t) = u_{a1}(t) + u_{a2}(t) = U_m \sin(\alpha t) + U_{n5} \sin(5\alpha t), \]  
(17)

\[ u_b(t) = u_{b1}(t) + u_{b2}(t) = U_m \sin(\alpha t + \frac{2\pi}{3}) + U_{n5} \sin(5\alpha t + \frac{2\pi}{3}), \]  
(18)

\[ u_c(t) = u_{c1}(t) + u_{c2}(t) = U_m \sin(\alpha t + \frac{2\pi}{3}) + U_{n5} \sin(5\alpha t + \frac{2\pi}{3}), \]  
(19)

And for currents:

\[ i_a(t) = i_{a1}(t) + i_{a2}(t) = I_m \sin(\alpha t - \varphi_a) + I_{n5} \sin(5\alpha t - \varphi_a), \]  
(20)

\[ i_b(t) = i_{b1}(t) + i_{b2}(t) = I_m \sin(\alpha t - \frac{2\pi}{3} - \varphi_a) + I_{n5} \sin(5\alpha t - \frac{2\pi}{3} - \varphi_a), \]  
(21)

\[ i_c(t) = i_{c1}(t) + i_{c2}(t) = I_m \sin(\alpha t - \frac{2\pi}{3} - \varphi_a) + I_{n5} \sin(5\alpha t - \frac{2\pi}{3} - \varphi_a), \]  
(22)

From (13) we can see that three phase instantaneous active power \( p_a(t) \) consists of sum of instantaneous powers from each phase. For the sake of brevity we will consider only phase a instantaneous power \( p_a(t) \) which can be defined by the following equation:

\[ p_a(t) = u_a(t)i_a(t) = u_{a1}(t)i_{a1}(t) + u_{a2}(t)i_{a2}(t) + +u_{a3}(t)i_{a3}(t) + u_{a5}(t)i_{a5}(t) \]  
(23)

(note: we should consider existence of others phase instantaneous powers \( p_b(t), \ p_c(t) \)). For simplification, we will separate parts of (23) created by harmonics with the same order of voltage and current from parts created by harmonics with different order of voltage and current that is:

\[ p_a(t) = x_a(t) + y_a(t) \]  
(24)

\[ x_a(t) = u_{a1}(t)i_{a1}(t) + u_{a2}(t)i_{a2}(t) \]  
(25)

\[ y_a(t) = u_{a3}(t)i_{a3}(t) + u_{a5}(t)i_{a5}(t) \]  
(26)

From (17), (20) and (25) we may derive:

\[ x_a(t) = \frac{U_m I_m}{2} (\cos\varphi_a - \cos\varphi_a - \cos(2\alpha t)) + \]

\[ + \frac{U_m I_{n5}}{2} (\cos\varphi_a - \cos\varphi_a + \cos(10\alpha t)) \]  
(27)

Part \( x_a(t) \) of instantaneous power \( p_a(t) \) contains of two terms. The first participate on production of active power \( P \) and the second participate on production reactive power \( Q \) for phase a. Parts in first square brackets represent instantaneous active power. Active power \( P \) of phase a can be defined as average value of this part over one period of fundamental component. Also can be by defined as average value of total phase instantaneous power \( p_a(t) \), because average values of instantaneous reactive and distortion powers over one period are zero. In general can be active power \( P \) expressed by:

\[ P_{1ph} = AVG(p_{1ph}(t)) = \frac{1}{T} \int_0^T p_{1ph}(t)dt \]  
(28)

where \( p_{1ph}(t) \) is one phase instantaneous power. Parts in second square brackets (27) represent instantaneous reactive power for phase a. Reactive power \( Q \) can be defined as a sum of maximal values of instantaneous reactive powers of each harmonic (with the same order of voltage and current).

\[ Q_{1ph} = \sum_{i=1}^{5} \text{MAX} \left[ \frac{U_m I_m}{2} \sin\varphi_i \sin(i \cdot 2\alpha t) \right] \]  
(29)

By defining \( y_a(t) \) as a sum of instantaneous active and reactive power, we connect function \( y_a(t) \) (26) with properties of distortion power (harmonics distortion). Definition of distortion power derives from interaction with different order harmonic components of voltage and current (so-called cross combination of voltage and current harmonics). That is consequential from function \( y_a(t) \). From (17), (20) and (26) we may derive the following equation:

\[ y_a(t) = \frac{U_m I_m}{2} (\cos(4\alpha t - \varphi_a) - \cos(6\alpha t - \varphi_a)) + \]

\[ + \frac{U_m I_{n5}}{2} (\cos(4\alpha \varphi_a) - \cos(6\alpha t - \varphi_a)) \]  
(30)

On the basis of the definition of apparent power \( S \) (2) and from P (28), Q (29) can be obtained value of
distortion power \( D \) for phase \( a \). Previous equations describe instantaneous power of phase \( a \). We can obtain equivalent equations and waveforms for others phases \( (x_b(t), x_c(t) \) and \( y_b(t), y_c(t) \) with the respect of phase shift (see also fig.4, 5 and 6). Thus, from one phase instantaneous power we can define properties of active, reactive and distortion power.

If we will sum each phase instantaneous powers \( p_a(t), p_b(t) \) and \( p_c(t) \) (equation (13)) then oscillating parts of \( x_a(t), x_b(t) \) and also parts of \( y_a(t), y_b(t) \) a \( y_c(t) \) with 40 frequency will cancel each others. This is given by following equation:

\[
x(t) = x_a(t) + x_b(t) + x_c(t) = 3\frac{U_{m1}}{2} \cos \varphi_1 + \frac{3}{2} \frac{U_{m5}}{2} \cos \varphi_5
\]

\[
y(t) = y_a(t) + y_b(t) + y_c(t) = -3\left(\frac{U_{m1}}{2} \cos(6\alpha - \varphi_3) + \frac{U_{m5}}{2} \cos(6\alpha - \varphi_5)\right)
\]

Fig. 4, 5 and 6 show examples of this functions and three phase active power \( p(t) \), included theirs phase parts during one period of fundamental harmonic (valid for following parameters: \( U_{m1}=100V, U_{m5}=10V, I_{m1}=10A, I_{m5}=5A, \varphi_1=\pi/4 \) a \( \varphi_5=\pi/4 \)).

\[
p(t) = x(t) + y(t) = p_{dc} + p_{ac}
\]  

We can see that \( p_{dc} \) represents three phase active power \( P_{3ph} \). It can be also defined as triple of one phase active power, defined by (28). AC component \( p_{ac} \) represents interaction between harmonics with different order of voltage and current (distortion power created by harmonics). From (31) it is clear that DC component \( p_{dc} \) of \( p(t) \) in this case of non-harmonic power supply is also created by 5.harmonic components of voltage and current. This fact eliminates the use of pq theory in reference current calculation of shunt APF in circuits with non-harmonic power supply (see also part of reference current calculation).

In paper [4] author intends if it is correct to use word “instantaneous” in pq theory. His reasoning was based on using the theory of the currents’ physical components. There are some examples: “According to pq theory can occur instantaneous reactive currents even if a load has zero reactive power (purely resistive load), moreover this current can be non-sinusoidal in circuit with a sinusoidal supply voltage and no source of current distortion in load.” To support this idea we can add some question with example. Can instantaneous active power during one period of fundamental harmonic contain two polarities? As an example can help fig.7, it is more theoretical than practical example, because in this case is amplitude of current 5.harmonic bigger than fundamental (consider circuit on fig. 3 and valid parameters: \( U_{m1}=100V, U_{m3}=10V, I_{m1}=5A, I_{m3}=10A, \varphi_1=\pi/4 \) a \( \varphi_3=\pi/4 \)).

Fig. 7 shows that in certain time load returns energy back to source. Behavior like this is usually connected with reactive or distortion (harmonic) power. In this case negative part of instantaneous active power is caused by harmonics interaction (amplitude of \( p_{ac}(t) \) that is distortion power (note: we still consider balanced voltage and current system, i.e. power created by unbalanced is neglected)). Total instantaneous active power for balanced three phase system should be constant and equal to triple of one phase active power, that is only DC component of “instantaneous active power” defined by pq theory.

Instantaneous reactive power defined by (12) has not any physical meaning, because is represented by non physical voltage system (16). Therefore authors of pq theory named this quantity as instantaneous imaginary power. In any case we would obtain similar results like before with “instantaneous active power” if we will use similar derivation. That means resultant instantaneous imaginary power would contain DC and AC components i.e.:

\[
q(t) = q_{dc} + q_{ac}
\]
Value of DC component would be equal to triple of reactive power \( Q_{abc} \) defined by (29) for one phase system. Otherwise for definition instantaneous imaginary power by (12) is DC component \( q_{dc} \) equal to its average value during one period of fundamental harmonic. It is not correct definition of reactive power in physical meaning. The AC component \( q_{ac} \) similarly like in case of “instantaneous active power” would represent interaction between unequal harmonics of voltage (defined by (16)) and current.

5. CONTROL STRATEGY OF APF, REFERENCE CURRENT CALCULATION

In following part we will consider sinusoidal balanced system of line voltages. Reference is the most important part of APF control. In fig.8 is shown block diagram of APF reference current calculation. The calculation is based on pq theory. Inputs of the calculation are phase voltages \((v_a, v_s, v_r)\) and phase load currents \((i_{aL}, i_{sL}, i_{cL})\). After transformation \((abc \rightarrow \alpha\beta)\), the \(\alpha\beta\) components of voltage and load current are the inputs of the block of instantaneous active and reactive power calculation defined by pq theory, equations (11) and (12). As it has been explained before, both of instantaneous powers contain DC and AC components. Shunt APF should inject to the nonlinear load current which consists of harmonics. It is not correct definition of reactive power like in case of “instantaneous active power” would contain not only active power of fundamental harmonic, but also some other parts (depend on harmonic spectrum of supply voltage and current, e.g.: in equation (30) with 5th harmonic component). That means bigger amplitude of current fundamental harmonic and then mistake in reference current calculation (see also simulation results).

![Fig. 8 Reference current calculation block diagram](image)

Reference current in \(\alpha\beta\) coordinates is transferred by inverse transformation \((\alpha\beta \rightarrow abc)\) to abc reference frame. The hysteresis-band controller is used for the current control. The reference current wave is compared with the actual phase filter current wave. As the error exceeds a prescribed hysteresis band, the upper switch is turned off and the lower switch is turned on. As a results, the output voltage transitions from 0.5 to -0.5 \(V_{dc}\), and the current starts decay. As the error crosses the lower band limit, the lower switch is turned off and the upper switch is turned on.

6. SIMULATION RESULTS

In this section, it is given the simulating results based on theory explained above. It was considered balanced voltage and current system and two possibilities of supply voltage, sinusoidal and non-sinusoidal. Three phase full bridge rectifier was used as a non-linear load. The following parameters were used: three phase balanced voltage system with
\( U_d = 325 \text{V}, f = 50\text{Hz} \), three phase full bridge rectifier with RL load \( R = 56.3\Omega \), \( L = 10\text{mH} \), active filter with voltage on DC bus \( V_d = 700\text{V}, C_d = 370\text{uF} \) and filter inductance \( L_d = 10\text{mH} \).

In fig. 10 are shown waveforms of line voltages, line currents, load currents and filter currents for a sinusoidal supply voltage. As it can be seen line current are almost sinusoidal and in phase with line voltage. The total harmonic distortion of line current is \( \text{THD}_L = 6\% \) compared with load current \( \text{THD}_N = 30\% \).

\[ U = 325\text{V}, f = 50\text{Hz}, \text{three phase full bridge rectifier} \]

8. ACKNOWLEDGEMENTS

This research project has been supported by a Marie Curie Early Stage Research Training Fellowship of the European Community's Sixth Framework Programme under contract number MEST-CT-2004-504243 Electrical Energy Conversion and Condition.

REFERENCES


