PID, 2-DOF PID and Mixed Sensitivity Loop-Shaping Based Robust Voltage Control of Quadratic Buck DC-DC Converter

Fateh OUNIS, Noureddine GOLEA

Electrical Engineering Department, Sciences and Applied Sciences Faculty, Larbi Ben M’hidi University, 04000 Oum El Bouaghi, Algeria
ounisfateh_01@yahoo.fr, nour_golea@yahoo.fr

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Abstract. DC-DC Quadratic Buck Converter (QBC) is largely used in applications where the high step-down conversion ratio is required. In the practical implementation, QBC is subject to uncertainties, disturbance, and sensor noise. To address the QBC control problems, a two-degree of freedom PID (2-DOF PID) is designed in the robust control framework. Further, for comparison purpose, a one-degree of freedom PID (1-DOF PID) and mixed sensitivity loop-shaping (MS-LS) controller are also proposed. Considering QBC parasitic components, the QBC small-signal transfer function is derived based on a practical approach. Sensitivity functions are used to specify the desired design requirements, and non-smooth optimization is used to tune both PID’s parameters. The three control structures are implemented and tested in the Matlab/Simulink environment. As attested by simulation results, the 2-DOF PID exhibits a better regulation accuracy with enhanced robust stability and robust performance for a wide range of supply voltage/load variation and sensor noise effect.

Keywords
DC-DC converters, mixed sensitivity, loop-shaping, PID control, quadratic buck converter, robust control.

1. Introduction

DC-DC converters are key elements in power energy modulation and conversion. Basic DC-DC converters, such as buck, boost and buck-boost, are widely used in various fields of technology [1]. Recently, new applications, such as LED lamps, microprocessors, portable devices and GPS, require very low dc voltages and they operate at very high currents. Such applications require converters with low ripples in the voltage and current and high efficiency in order to achieve precise output voltage regulation against parameter, line and load disturbances. Basic step-down converters are not suitable for high step-down voltage conversion since operating at a small duty ratio affects the converter dynamic performance and cause asymmetry in the on and off times of the switches. Moreover, very small duty ratio limits the converters switching frequency and increases peak switch current that leads to more switching losses, severe reverse-recovery problems and converter’s efficiency degradation [2]. Some cascade interconnected power converters structures were developed to face this problem. However, a notable disadvantage of cascaded converters is that the overall efficiency is reduced by losses in switching devices. To improve overall efficiency, Quadratic Buck Converters (QBC) were developed in [3], [4], [5] and [6]. QBC is designed based on cascade connection of two buck converters and has only one active switching device. The DC conversion ratio is the product of the conversion ratios of the two single buck converters. QBC operates at higher switching frequencies with wide load range and achieves an improved step-down conversion ratio. The efficiency is also enhanced since only one active switch is used [5] and [7].

As QBC exhibits complex nonlinear dynamics subject to parameters uncertainties and input/load variations, control loops must be introduced to guarantee stability and operating performance. Several QBC control techniques such as linear state feedback, feedback linearization, sliding mode control and passivity based control, were presented in [10]. In [11], QBC nonlinear control scheme is proposed. QBC with LC input filter and damping control is developed in [12].
Robust QBC control based on identified Hammerstein model is proposed in [13]. Average current-mode control for the QBC is proposed in [13], where the outer voltage control loop bandwidth is limited by inner current control loop bandwidth. In [14], robust control for QBC is designed based on Kharitonov’s theorem and D-stability concept. To ensure robust output regulation, robust state feedback stabilizer with saturated inner model, is proposed in [16]. In [17], inner current loop PI parameters are selected from QBC large-signal model, and outer voltage loop is controlled using a conventional PI regulator. To ensure robustness in the presence of disturbances and uncertainties, $H_{\infty}$ based control is investigated in [18].

In this paper, 2-DOF PID is developed to solve the QBC robust control problem. For the comparison, a 1-DOF PID and MS-LS control are also proposed. Taking the parasitic components into account, the QBC small signal transfer function from output voltage to control signal is derived. Robust performance requirements are defined using the same weighting functions for both PID and by another set of weighting functions for MS-LS control. Contrary to MS-LS control and 1-DOF PID, the 2-DOF PID provides a wide range of the crossover frequency to specify response time-performance compromise. The non-smooth approach presented in [19] is used to tune both PID’s parameters. Based on the model reduction methods, MS-LS controller is reduced from 8 to 5, and the reduced version is presented and used in simulations. Simulation results illustrate the 2-DOF PID in term of accuracy and stability robustness.

The remainder of this paper is organized as follows. Section 2 presents the QBC nominal transfer function computation. The MS-LS control is developed in Section 3. PID controller’s design is provided in Section 4. The three controllers’ robustness analysis is established in Section 5. Simulation results are shown in Section 6. Concluding remarks are given in Section 7.

2. QBC Nominal Model

As a first step for the control design, the QBC open loop small-signal control-to-output voltage transfer function should be established. The objective can be reached using analytical modeling and averaging techniques [20] and [21]. In this work, a practical approach is adopted. Based on QBC parameters and operating point given in App. A a Simulink implementation of the open loop excitation is realized (Fig. 1). The PWM control signal duty cycle is adjusted to get the desired output voltage level. Taking into account the parasitic components, the QBC discrete-time transfer function $G(z) = B(z)/A(z)$ is assumed of 6th order. Hence, applying the Steiglitz-McBride recursive identification method [22] on converter averaged input/output signals for 5 iterations, yields the estimated transfer function:

$$B(z) = 0.7108 - 1.6654 z^{-1} + 1.9263 z^{-2} - 1.6483 z^{-3} + 1.2154 z^{-4} - 0.5151 z^{-5},$$

$$A(z) = 1 - 4.829 z^{-1} + 10.174 z^{-2} - 11.9259 z^{-3} + 8.1753 z^{-4} - 3.1028 z^{-5} + 0.5094 z^{-6}. \quad (1)$$

Further, using the numerical algorithm proposed in [23], the equivalent continuous-time transfer function $G(s) = N(s)/D(s)$ is given by:

$$N(s) = 544300 s^9 + 1.618 \cdot 10^{11} s^8 + 2.184 \cdot 10^{11} s^7 + 1.524 \cdot 10^{23} s^6 + 7.774 \cdot 10^{29} s^5 + 3.487 \cdot 10^{34},$$

$$D(s) = s^6 + 674400 s^5 + 7.804 \cdot 10^{11} s^4 + 3.354 \cdot 10^{17} s^3 + 1.502 \cdot 10^{23} s^2 + 2.677 \cdot 10^{28} s + 2.342 \cdot 10^{33}. \quad (2)$$

The $G(s)$ Bode plot is shown in Fig. 2. It is clear that QBC has two second-order filters with high quality-factor $Q$, which depends on the selected circuit values. All poles and zeros are located on the right half of the s-plane as shown the Fig. 3. The right half plane zeros are responsible for the excessive phase lag in the ideal case. The Equivalent Series Resistances (ESR) provide some damping into the system, which is beneficial as it will ease feedback control design.

3. MS-LS Control Design

A diagram of the control design is shown in Fig. 4 where $G$ is the quadratic buck converter transfer function. $W_1$, $W_2$ and $W_3$ are the performance, control and noise weighting functions, respectively. Further, $w$ denote input signals, $z$ output vector that includes...
both performance and robustness measures, \( v \) is the vector of measurements available to the controller \( K \) and \( u \) the control signal. In the context, the necessary definitions are given by:

\[
S(s) = (I + G(s)K(s))^{-1},
\]

\[
T(s) = G(s)K(s)(I + G(s)K(s))^{-1},
\]

where \( S(s) \) is the sensitivity function and \( T(s) \) is the complementary sensitivity function. The generalized closed loop transfer function is given by:

\[
T_{zw} = \begin{bmatrix} W_1S & W_2RS \\ W_3RS & W_4T \end{bmatrix},
\]

where \( R(s) = K(s)(I + G(s)K(s))^{-1} \). In this mixed problem, the control objective is to design a stable controller that minimizes the norm of the generalized transfer function \( T_{zw} \) such that:

\[
||T_{zw}||_\infty < 1.
\]

### 3.1. Weighting Functions Selection

The closed loop performance of the system is largely dependent on the shape of the weighting function. The weight function \( W_1 \) specifies the control performance and \( W_1 \) is selected according to methodology suggested by Zhou [24],

\[
W_1 = \frac{s/M_s + w_s}{s + w_s e_s},
\]

where \( e_s \) is the maximum allowed steady-state offset fixed to \( e_s = 0.001 \), \( w_s \) is the desired bandwidth fixed to \( 6 \cdot 10^3 \) rad\(\cdot\)s\(^{-1}\) and \( M_s \) is the sensitivity peak (typically \( M = 1.6 \)). Therefore,

\[
W_1 = \frac{0.625(s + 9600)}{s + 6}.
\]

In order to avoid impulsive input effect on the converter, \( W_2 \) is chosen as:

\[
W_2(s) = 0.01.
\]

\( W_3 \) is used to shape the complementary sensitivity function \( T \), and thus it must be large at high frequencies. Hence \( W_3 \) is chosen as:

\[
W_3 = \frac{s + w_b/M_b}{e_b s + w_b}.
\]
value 0.001 is selected for the parameter $e_b$. In order to limit the closed loop bandwidth, the parameter $M_b$ is fixed as 1.6 and $w_0$ is fixed to $10^4$ rad/s$^{-1}$. Then:

$$W_3 = \frac{1000(s + 6250)}{s + 10^4}. \quad (11)$$

In order to adopt a unified solution procedure, the above matrix inequality Eq. (6) can be recast into a standard configuration as in Fig. 4. This can be obtained by using the Linear Fractional Transformation (LFT), and the generalized plant $P$ is obtained by grouping signals into sets of external inputs, outputs, input to the controller and output from the controller, which yields:

$$
\begin{bmatrix}
\dot{z} \\
\tau
\end{bmatrix} =
\begin{bmatrix}
W_1 & -W_1G \\
0 & W_2G \\
0 & W_3G \\
1 & -G
\end{bmatrix}
\begin{bmatrix}
z \\
\tau
\end{bmatrix}, \quad (12)
$$

where $r = w$ is the reference voltage, $z = [z_1 \ z_2 \ z_3]^T$ is the output signals vector, is the control signal and is the controlled QBC output voltage. $W_1$, $W_2$ and $W_3$ are the weighting functions described by Eq. (8), Eq. (9) and Eq. (11), respectively.

Based on the above configuration, the generalized plant can be built up, and consequently the controller can be calculated using Matlab robust control toolbox. Hence, the obtained 8th order controller is:

$$K(s) = \frac{N_K(s)}{D_K(s)}, \quad (13)$$

with

$$N_K(s) = 5.504 \cdot 10^6s^7 + 5.875 \cdot 10^{13}s^6 + 4.141 \cdot 10^{19}s^5 + 4.48 \cdot 10^{25}s^4 + 1.929 \cdot 10^{11}s^3 + 8.414 \cdot 10^{36}s^2 + 1.486 \cdot 10^{42}s + 1.289 \cdot 10^{47},$$

and

$$D_K(s) = s^8 + 5.444 \cdot 10^9s^7 + 5.525 \cdot 10^{15}s^6 + 2.3 \cdot 10^{22}s^5 + 1.718 \cdot 10^{28}s^4 + 8.868 \cdot 10^{33}s^3 + 5.922 \cdot 10^{39}s^2 + 2.5 \cdot 10^{44}s + 1.5 \cdot 10^{45}.$$

The obtained controller Eq. (13) has high order, and can be further, reduced by examining $K(s)$ Hankel singular values $\sigma_i$. Hankel singular values based model reduction routines are grouped by the types of error bound. In Balanced Truncation (BT) and related methods, an error bound is a measure of how close the reduced order controller $K_r(s)$ is to the original system and is computed based on the infinity norm of the additive error,

$$||K(s) - K_r(s)||_{\infty} = \sum_{i=1}^{n} (\sigma_i), \quad (14)$$

with

$$K_r = \frac{N_{K_r}(s)}{D_{K_r}(s)}. \quad (15)$$

The basic idea of BT relies on balancing the two controllers’ controllability Gramian and operability Gramian $[23]$. The Hankel singular values plotted in Fig. 5 are used to decide which states of the controller can be safely discarded. To achieve at least 1% relative accuracy, the lowest-order controller $K_r(s)$ should be compatible with the desired level of accuracy chosen to be 5.

![Hankel singular values of $K(s)$](image)

Fig. 5: Hankel singular values of $K(s)$.

The function "reduce" is the gateway to all model reduction routines available in the toolbox MATLAB. We use the default, square-root balance truncation ("balancmr") option of "reduce" as the first step. This method uses an "additive" error bound for the above described reduction method, meaning that it tries to keep the absolute approximation error uniformly small for all frequencies.

The error bound for additive-error algorithms is defined as:

$$||K(s) - K_r(s)||_{\infty} = 2(\sigma_6 + \sigma_7 + \sigma_8) = 0.0088, \quad (16)$$

which yields:

$$N_{K_r}(s) = 1.045 \cdot 10^{4}s^4 - 3.36 \cdot 10^{9}s^3 + 5.937 \cdot 10^{15}s^2 - 1.393 \cdot 10^{21}s + 6.808 \cdot 10^{26},$$

$$D_{K_r}(s) = s^5 + 2.848 \cdot 10^{5}s^4 + 4.018 \cdot 10^{12}s^3 + 1.128 \cdot 10^{17}s^2 + 1.45 \cdot 10^{24}s + 7.464 \cdot 10^{24}.$$

According to condition Eq. (16), it is necessary that the magnitude response of $S$ lies below the magnitude response of $W_1^{-1}$ in the whole frequency range, and
the magnitude response of $T$ should lie below the response of $W^{-1}$. Figure 6 shows that these conditions are verified using the reduced controller.

![Singular Values](image)

**Fig. 6:** MS-LS control design results.

### 4. 1-DOF PID and 2-DOF PID Controllers Design

In the following section, the control system structure of Fig. 7 is adopted, where $C(s)$ is the standard controller, $C_F(s)$ the input filter, and $G$ is the converter transfer function.

![2-DOF PID structure](image)

**Fig. 7:** 2-DOF PID structure.

The standard PID controller is used with the transfer function:

$$C(s) = K_P + \frac{K_I}{s} + \frac{K_D s}{T_F s + 1}$$  \hspace{1cm} (17)

with the proportional gain $K_P$, the integrator gain $K_I$, the derivative gain $K_D$, and the derivative filter time constant $T_F$.

#### 4.1. 2-DOF PID Controller

The output signal of a 2-DOF regulator is defined as:

$$u(s) = K_P e_p + K_I e_I + K_D e_D,$$  \hspace{1cm} (18)

where

$$\begin{align*}
e_p &= br(s) - y(s) \\
e_I &= \frac{1}{s} \left(r(s) - y(s)\right) \\
e_D &= (cr(s) - y(s))
\end{align*}$$  \hspace{1cm} (19)

where $b$, $c$ are weight parameters for proportional term and derivative term, respectively. The 2-DOF controller can be transformed into a 1-DOF controller, if $b$ and $c$ are selected to be equal to 1. To formulate the closed loop transfer function, the output of controller Eq. (19) is rewritten as:

$$u(s) = (C_F(s)r(s) - y(s))C(s).$$  \hspace{1cm} (20)

The closed loop control system output to the perturbation is given by:

$$y(s) = \frac{C_F(s)G(s)}{1 + C(s)G(s)} r(s) + \frac{G(s)}{1 + C(s)G(s)} d(s).$$  \hspace{1cm} (21)

The system closed loop transfer function is defined as:

$$T(y)(s) = C_F(s)\frac{C(s)G(s)}{1 + C(s)G(s)}.$$  \hspace{1cm} (22)

The parameters $\{K_P, T_I, T_D, b, c\}$ are obtained considering the targeted specifications.

#### 4.2. Frequency Specifications

To ensure that the output voltage tracks the reference with a desired response time and tracking error, transfer function is used to specify the maximum frequency-domain tracking error:

$$e_{\text{max}} = A_e s + \omega_c D_e \frac{s + \omega_c}{s},$$  \hspace{1cm} (23)

where $\omega_c = 2/t_s$ ($t_s$ is the settling time) is the tracking bandwidth, $D_e$ is the maximum relative steady-state error and $A_e$ is the peak relative error across all frequencies. For the QBC, we set $D_e = 0.001$, $A_e = 1$ and $\omega_c = 10^3 \text{rad-s}^{-1}$, since the open loop-gain should be high within the control bandwidth. To ensure good disturbance rejection, the minimum loop gain profile is chosen as:

$$W_s = \frac{0.03\omega_c}{s}.$$  \hspace{1cm} (24)

To ensure insensitivity to measurement noise, the open loop gain should be less than 1 outside the control bandwidth, so the maximum loop gain profile is chosen as:

$$W_T = \frac{0.3\omega_c}{s}.$$  \hspace{1cm} (25)

Using software provided by Matlab, the above requirements are converted into normalized scalars functions $f(x)$ and $g(x)$ such as:

$$g(x) = \left\|\frac{1}{e_{\text{max}}} (T(s, x))\right\|_{\infty}.$$  \hspace{1cm} (26)
\[ f(x) = \begin{vmatrix} W_s S_a \\ W_T^{-1} T_a \end{vmatrix}_\infty, \]  

(27)

where \( T(s, x) = \frac{L}{1 + L} \) is the output complementary sensitivity function, \( L(s, x) \) is the open-loop response being shaped, \( T_a = D^{-1} T D \) is the scaled output complementary sensitivity function, \( S_a = D^{-1} \left[ (1 + L(s, x)) \right]^{-1} D \) is the scaled output sensitivity function, \( x \) is the vector of free (tunable) parameters \( K_P, K_I \) and \( K_D \).

Then, determining the PID parameters is equivalent to solving the optimization problem:

\[ \min_x \max \left( a f(x), g(x) \right), \]  

(28)

where \( a > 0 \) is a parameter weighting the subproblems importance in order to get the most optimal solution for the optimization problem. Nonsmooth optimization algorithms [26] and [27] are used to solve the QBC converter control problem. According to the required desirable performance, the ultimate PIDs parameters are achieved as follows:

1-DOF: \( K_P = 0.00865, K_I = 230, K_D = -8.03 \cdot 10^{-5}, \)

2-DOF: \( K_P = 0.00824, K_I = 298, K_D = 1.68 \cdot 10^{-5}, b = 0.00015, c = 0.226. \)

5. Robust Analysis

We can test the robustness properties of the three controllers by executing the proper \( \mu \) tests for the QBC uncertain feedback system shown in Fig. 8 where the dashed box represents the QBC real transfer function \( G_{unc}. \) The transfer functions \( W_{del} \) and \( \Delta_G \) parameterize the multiplicative uncertainty at the converter input. The transfer function \( W_{del} \) is assumed known, and the transfer function \( \Delta_G \) is assumed to be stable and unknown, except for the norm condition \( ||\Delta_G||_\infty < 1. \) The uncertainty weight \( W_{del} \) is described as:

\[ W_{del}(s) = \frac{100s + 7.035 \cdot 10^7}{s + 7.035 \cdot 10^8}. \]  

(29)

Figure 9 compares the upper bounds of the structured singular values, for the robust stability analysis of the closed-loop systems with the three controllers (1-DOF PID, 2-DOF PID and MS-LS control). To achieve robust stability, it is necessary that the \( \mu \)-values are less than 1 over the frequency range [28] and [29]. It is clear that the controllers achieve a robust stability. The best robustness is obtained by the MS-LS controller.
The robust performance is achieved if and only if for each frequency computed for the closed-loop frequency response is less than 1. The robust performance tests for the three controllers are shown in Fig. 10. Again, the MS-LS controller shows large $\mu$ values over the low-frequency range.

6. Simulation Results

As shown in Fig. 11, the three controllers designed in the above section are implemented in Matlab/Simulink. Note that MS-LS controller $K_r(s)$ is implemented using the state space realization $(A, B, C, D)$. To compare the three controllers’ performances and robustness, the following tests are performed.

6.1. Set Point Tracking

The QBC response for a 10 V constant reference voltage is shown in Fig. 12. It can be observed that QBC settling time is 1 ms for the MS-LS control and 2-DOF PID controls, which is faster as compared to the 1.6 ms settling time for the 1-DOF PID control. Another aspect is that MS-LS control exhibits an overshoot of 16.5 %; while the overshoot for the 1-DOF PID and 2-DOF PID is of the order of 15.5 %.

In addition, when the reference input voltage changes from 10 to 12 V, as shown in Fig. 13, an oscillatory behavior is observed for MS-LS control. Comparison to that PID controllers provide a more damped behavior, in response to the reference voltage increase or decrease, as can be noticed from Fig. 14 and Fig. 15.

6.2. Load Variation

A load variation of 100 % (from 10 to 20 $\Omega$) is introduced between 10 and 30 ms. QBC response shown in Fig. 16 indicates that all control methods provide almost the same performance. The output voltage exhibits an undershoot of 6 % and an overshoot of 8 %.
6.3. Supply Voltage Variation

A voltage drop of 3 V is introduced in the supply voltage between 5 and 10 ms. Figure 17 shows that the three control methods obtained almost the same stabilizing time with same undershoot (about 7% at 5 ms). At 10 ms, similar overshoot (35%) is observed for the three control methods. However, at 10 ms, PID control methods exhibit a null undershoot compared to 3% for MS-LS control.

6.4. Disturbance Rejection

The supply voltage is perturbed by a sinusoidal component of 100 Hz frequency and 2 V peak-to-peak amplitude. Further, sensor white noise, of $10^8$ rad·s$^{-1}$ frequency and $10^{-5}$ power, is assumed to be superposed on the output voltage. Figure 18 shows that
2-DOF PID controller has better disturbance rejection than 1-DOF PID and MS-LS controllers in this band of frequencies.

7. Conclusion

A 2-DOF PID controller is proposed, designed and simulated for the quadratic buck converter. For comparison purpose, 1-DOF PID and MS-LS control controllers are also tested. Even if MS-LS control shows a faster response, 2-DOF PID provides more damped and accurate response. Further, under perturbations and uncertainties, 2-DOF PID control exhibits better robustness in performance and stability compared to MS-LS control. Another practically important advantage of the 2-DOF PID is a lower structure complexity compared to MS-LS control.

References


About Authors

Fateh OUNIS was born in Oum El Bouaghi, Algeria. He received the Master degree in electrical engineering from Oum El Bouaghi University, in 2009. Actually, he is finalising Ph.D. thesis. His research interests are power converters design and control.

Noureddine GOLEA received his Doctorate degree in 2001 from Batna University, Algeria. Currently, he is a full Professor in Electrical Engineering Department at Oum El Bouaghi University, Algeria. His research interests are non-linear control, intelligent control and power systems control.

Appendix A

Quadratic Buck Converter Parameters

<table>
<thead>
<tr>
<th>Circuit parameters</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Input voltage $E$</td>
<td>24 V</td>
</tr>
<tr>
<td>Reference voltage $v_2$</td>
<td>10 V</td>
</tr>
<tr>
<td>Switching frequency $f$</td>
<td>110 kHz</td>
</tr>
<tr>
<td>$L_1$</td>
<td>39 $\mu$H</td>
</tr>
<tr>
<td>$L_2$</td>
<td>27 $\mu$H</td>
</tr>
<tr>
<td>$C_1$</td>
<td>16 $\mu$F</td>
</tr>
<tr>
<td>$C_2$</td>
<td>18 $\mu$F</td>
</tr>
<tr>
<td>$r_{L_1}, r_{L_2}, r_{C_1}, r_{C_2}$</td>
<td>0.25 $\Omega$</td>
</tr>
<tr>
<td>Load $R$</td>
<td>10 $\Omega$</td>
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