

# UPPER CAPACITY BOUNDS OF MIMO WIRELESS SYSTEMS THROUGH FADING CHANNELS

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**Abstract.** *This paper investigates the upper capacity bounds of MIMO systems with correlation and antenna selection techniques in general fading environments. With Antenna Selection techniques, the increased hardware complexity due to multiple antennas and large number of RF chains can be reduced to a substantial amount, retaining the diversity benefits of MIMO systems. The channel Correlation also affects the capacity of MIMO fading channels. Hence, to evaluate the upper bounds of capacity through fading channels, performance of MIMO systems is exemplified under Nakagami-m and Rayleigh fading channels while considering that the channel characteristics are known at a transmitter. The obtained results give an assessment to the better understanding to the effect of antenna selection and correlation on the capacity of MIMO channels, and how they can be used in different fading environments.*

## Keywords

*Antenna selection, capacity, correlation, fading channels.*

## 1. Introduction

Due to constantly increasing demand of high data transmissions in wireless communication systems, the multi-antenna systems have been actively investigated and successfully deployed for the emerging wireless broadband networks. The channel capacity of a multi-antenna system is increased as compared to a conventional single antenna system, without any additional transmit power or spectral bandwidth. These multi-antenna systems are commonly known as MIMO (Multiple Input and Multiple Output) systems. They are

a part of modern wireless communication standards such as IEEE 802.11n (Wi-Fi), MBWA, HSPA, 3GPP LTE, WiMAX-m and LTE Advanced [1]. The multiple antennas in MIMO systems can be exploited as Antenna Diversity or Spatial Multiplexing. Antenna Diversity is used in wireless systems to combat the effects of fading, and is intended to transmit the same information-bearing signals over multiple antennas at the transmitter or to receive them from multiple antennas at the receiver; and thereby improve the reliability of transmission [2]. For MIMO systems having  $N_T$  transmit and  $N_R$  receive antennas, a diversity order of  $N_T N_R$  can be achieved [3]. The Spatial Multiplexing is the other way of exploiting MIMO systems for the transmission of several parallel data streams, and increasing the capacity of wireless systems. The separately encoded data streams are transmitted over multiple transmit antennas and received separately by multiple antennas at the receiver and thus increases the data rates [2]. Thus, the great potential for achieving high data rates and providing reliable communication has attracted massive attention. Motivated by huge opportunities [1], the capacity and performance of MIMO antenna systems in different fading channels is characterized with the reliance of 'Antenna Selection techniques' and 'Correlation'.

The major problem with MIMO systems is the additional high cost of RF (radio-frequency) modules which are required because of multiple antennas being used [4], [5]. The RF modules include a low noise amplifier (LNA), a frequency down-converter, and an analog-to-digital converter (ADC) [6] that are very expensive. In an effort to reduce the cost associated with the multiple RF modules, Antenna Selection Techniques are used to employ a smaller number of RF modules than the number of transmit and receive antennas. Blum and Winters [7] and Zhang [8] describe that the antenna selection technique could be used as a promising approach for reducing hardware complexity while re-

taining a high capacity of MIMO systems. Hiwale [9] investigates the impact of receive antenna selection on the capacity and error probability analysis of MIMO systems. The results with and without antenna selection of MIMO systems are compared and it is analyzed that the achieved capacity with receive antenna selection is close to the capacity of full complexity MIMO system. Molisch and Win [10] represent MIMO systems with reduced complexity where one link-end uses all the available antennas, while the other end chooses the  $L$  out of  $N$  antennas which maximizes the system capacity. The suboptimum antenna selection algorithm is discussed having smaller complexity of the order  $N^2$  as compared to optimum algorithm with a complexity of  $\binom{N}{L}$ .

The Channel Correlation has a significant impact on the performance of a MIMO system. Under the i.i.d. (independent and identically distributed) model where the channel matrix has i.i.d. zero-mean proper-complex Gaussian entries, the optimal input is an i.i.d. zero-mean proper-complex Gaussian vector. While the i.i.d. model facilitates the analysis, it is an ideal model representing a rich uniform scattering that rarely occurs in practice. It is hence of interest to study a more realistic model where the elements of the channel matrix are correlated. Saeed and Khatun [11] reviewed that the capacity of a multiple-input multiple-output system increases linearly with the number of antennas, given that the environment is rich scattering. However, this increase in capacity is substantially degraded if the MIMO antenna gains are correlated. Kiessling and Speidel [12] derive exact formulas for the calculation of ergodic capacity of a fully correlated MIMO channel in a flat Rayleigh fading environment and provide the negative impact of channel correlation on the ergodic capacity. Hanlen and Grant [13] express capacity in terms of an i.i.d. component and a correlated component which are then used to compare the correlated channel with the well-known i.i.d. channel. It is shown that the i.i.d. channel is optimal in terms of linear growth, providing the greatest increase in channel capacity, over the correlated channel.

Thus so far, only a small set of published literature investigates the antenna selection for MIMO wireless systems and thus only limited results are available on the performance analysis of antenna selection techniques in MIMO. In this paper, the performance of MIMO antenna selection techniques is described in different fading channels, along with the methods by which the best antennas are selected in an efficient manner. The pioneering work in the area of multi-antenna communications predicted remarkable spectral efficiency of MIMO wireless systems in independent and identically distributed (i.i.d.) Rayleigh fading. However, little subsequent work is concentrated on characterizing MIMO capacity under correlated fading.

In this paper, the analytical results for the capacity, such as ergodic capacity, are illustrated under correlated MIMO channels. The rest of this paper is organized as follows: The fading channel models are expressed in section 2. The capacity bound dependencies of Correlation and Antenna selection techniques on the MIMO fading channel are discussed in section 3. The simulation methodology and simulation environment are analyzed in section 4. followed by the results acquired in Nakagami-m and Rayleigh fading environment when the channel characteristics are known at the transmitter, in section 6.

## 2. Channel Model

In wireless systems, fading occurs due to multipath propagation or shadowing from obstructions affecting the wave propagation. When there is no LOS (line of sight) component between the transmitter and the receiver, the received signal resulting from the reflections, diffractions and scattering along the propagation paths, at the transmitter is a complex Gaussian random variable and follow the Rayleigh distribution. Considering this Gaussian random variable, the probability density function (pdf) of Rayleigh fading is given as [14]:

$$f_{Rayleigh}(r) = \frac{r}{\sigma^2} e^{-\frac{r^2}{2\sigma^2}} \quad (0 \leq r \leq \infty), \quad (1)$$

where  $\sigma$  is the RMS (root mean squared) value and  $\sigma^2$  is the average power of the received fading signal.

The multipath fading distribution is generally modeled with the Rayleigh distribution, but when fading is severe (NLOS), the Rayleigh model fall short to characterize the exact channel characteristics. Thus, a dominant model, named Nakagami-m model, is used to represent the channel. The significant application of the Nakagami-m channel model is its versatility to state other random channel distributions. Assuming  $r$  is a Nakagami random variable, the corresponding pdf is described as [15]:

$$f_{Nakagami-m}(r) = \frac{2r^{2m-1}\Omega}{\Gamma(m)\Omega^m} e^{-\frac{r^2}{\Omega}} \quad (0 \leq r \leq \infty), \quad (2)$$

where  $\Gamma(\cdot)$  is the Gamma function,  $\Omega = \frac{r^2}{m}$ ,  $r^2$  is the average received signal power, and  $m$  is the inverse normalized variance which satisfies the condition of  $m \geq \frac{1}{2}$ , describing the fading severity. The Nakagami-m channel model can also be used to approximate the one-sided Gaussian distribution ( $m = \frac{1}{2}$ ), Rayleigh distribution ( $m = 1$ ), Rician ( $m = 2$ ) and several other random distributions with the help of some appropriate one-to-one parameter mapping algorithms. When

$m \rightarrow \infty$ , the Nakagami- $m$  distributed fading channel will converge to a non-fading additive white Gaussian noise (AWGN) channel [14]. Nakagami- $m$  distributed fading channel exists in the literature for values of Nakagami- $m$  parameter,  $m = 0.5$  to 10. In addition, the importance of Nakagami- $m$  fading channel model is in the fact that it gives the widest span of the amount of fading (also called fading figure), among usual fading channel models.

### 3. Capacity Bound Dependencies

The fading channel challenges the system designers and engineers to establish a system that provides reliable communication between a transmitter and a receiver, as fading channel has a great influence on the upper capacity bound of MIMO wireless systems. The effect of Antenna Selection techniques and channel correlation on the capacity of MIMO wireless systems is described in the following subsections.

#### 3.1. Antenna Selection for MIMO

The advantage of MIMO systems is better performance achieved, without using additional transmit power or bandwidth extension. The capacity of MIMO systems increases linearly with  $\min(N_T, N_R)$ , where  $N_T$  and  $N_R$  are the numbers of transmit and receive antennas, respectively. However, the higher performance of MIMO systems comes at the expense of increased hardware requirements and computational complexity due to multiple RF chains required. MIMO systems with  $N_T$  transmit and  $N_R$  receive antennas require  $N_T N_R$  complete RF chains at the transmitter and receiver. Thus, natural concern in the implementation of MIMO systems is the increased hardware requirements.

In order to reduce the hardware cost and preserve the advantages of MIMO systems, the promising techniques are referred to as Antenna Selection Techniques. With these methods, the RF chains are optimally connected to the best subset of the transmitter/receiver antennas; means the best set of antennas is used, whereas the remaining antennas are not used. Thus a reduced number of RF chains can be employed at the transmitter/receiver and each chain can be optimally allocated to one of a larger number of antennas. Antenna selection can be simultaneously employed at the transmitter and receiver in a MIMO system. The system performance using antenna selection techniques is better than the full-complexity systems with the same number of antennas but without the selection. However, this superior performance is obtained by antenna selection at the cost of additional computational com-

plexity which grows linearly with  $(N, Q)$ , where  $N$  and  $Q$  are the total and selected number of antennas, respectively. A number of algorithms are developed for selecting the best possible antenna subset in MIMO wireless systems such as optimal and suboptimal antenna selection methods. Figure 1 shows MIMO wireless system with selecting  $Q$  antennas out of  $N_T$  transmit antennas, where  $Q > N_T$ . It means that the  $Q$  RF modules are selectively mapped to  $Q$  of  $N_T$  transmit antennas.

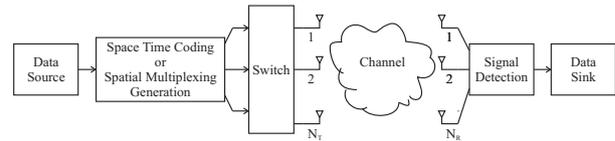


Fig. 1: MIMO systems with transmit antenna selection by using  $Q$  RF modules to support  $N_T$  transmit antennas [8].

Considering  $Q$  antennas being used among the  $N_T$  transmit antennas, the effective channel is represented by matrix  $\mathbf{H} \in \mathbf{C}^{N_R \times N_T}$  and the index of  $i^{th}$  selected column is represented by  $p_i$ , where  $i = 1, 2, \dots, Q$ . The corresponding effective channel could be modeled by  $N_R \times Q$  matrix and denoted by  $\mathbf{H}_{\{p_1, p_2, \dots, p_Q\}} \in \mathbf{C}^{(N_R \times Q)}$ . The space-time-coded or spatially-multiplexed stream  $x \in \mathbf{C}^{Q \times 1}$  is mapped into  $Q$  selected antennas as shown in Fig. 1 and the received signal  $y$  is represented as follows [6]:

$$y = \sqrt{\frac{E_x}{Q}} \mathbf{H}_{\{p_1, p_2, \dots, p_Q\}} x + z, \tag{3}$$

where  $z \in \mathbf{C}^{N_R \times 1}$  is the additive noise vector. It indicates that the channel capacity of the system depends on which transmit antennas are chosen as well as the number of transmit antennas that are chosen. The antenna selection techniques are discussed in the following subsections.

#### 1) Optimum Antenna Selection Technique

Antenna selection techniques are used to employ a smaller number of RF modules than the number of transmit/receive antennas. Thus, optimal antenna selection schemes are of great interest, where the best  $Q$  out of  $N_T$  antenna signals is chosen, downconverted and processed. It reduces the number of required RF chains from  $N$  to  $Q$  and significantly reduces the cost of establishing MIMO systems. In this section, the ‘‘Channel Capacity’’ is used as a design criterion for antenna selection.

Consider a set of  $Q$  transmit antennas is selected out of  $N_T$  transmit antennas so as to maximize the channel capacity. When the total transmitted power is limited by  $P$ , the channel capacity of the system using

$Q$  selected transmit antenna is represented as follows [8]:

$$C = \max \log_2 \det \left( I_{NR} + \frac{Ex}{\mathbf{Q}N_o} \mathbf{H}_{\{p_1, p_2, \dots, p_Q\}} \cdot \mathbf{R}_{xx} \mathbf{H}_{\{p_1, p_2, \dots, p_Q\}}^H \right) \text{ (bps} \cdot \text{Hz}^{-1}), \tag{4}$$

$$\mathbf{R}_{xx} \{p_1, p_2, \dots, p_Q\},$$

where  $\mathbf{R}_{xx}$  is  $\mathbf{Q} \times \mathbf{Q}$  covariance matrix. If equal power is allocated to all selected transmit antennas,  $\mathbf{R}_{xx} = I_Q$ , which yields the channel capacity for the given  $p_i$  as [6]:

$$C_{\{p_1, p_2, \dots, p_Q\}} \triangleq \log_2 \det \left( I_{NR} + \frac{Ex}{\mathbf{Q}N_o} \cdot \mathbf{H}_{\{p_1, p_2, \dots, p_Q\}} \mathbf{R}_{xx} \mathbf{H}_{\{p_1, p_2, \dots, p_Q\}}^H \right). \tag{5}$$

It corresponds to the optimal selection of  $P$  antennas for all possible antenna combinations and to maximize the system capacity, the antenna with the highest capacity is chosen as [6]]:

$$\{p_1^{opt}, p_2^{opt}, \dots, p_Q^{opt}\} = \arg \max C_{\{p_1, p_2, \dots, p_Q\}} \tag{6}$$

$$\{p_1, p_2, \dots, p_Q\} \in A_Q,$$

where  $A_Q$  represents a set of all possible antenna combinations with  $Q$  selected antennas. However, considering all the possible antenna combinations (in Eq. (6)) involves the enormous complexity, especially when the  $N_T$  is very large. Therefore, another method to reduce the complexity is developed as described in the next subsection which considers this particular issue.

### 2) Sub Optimal Antenna Selection

As mentioned in the previous subsection, optimal antenna selection in Eq. (6) may involve too much complexity depending on the total number of available transmit antennas. In order to reduce its complexity, it is required to resort to the sub-optimal method by which the additional antenna could be selected in Ascending Order of increasing the channel capacity. Thus, one antenna with the highest capacity is first selected as:

$$p_1^{subplot} = \arg \max_{\{p_1\}} C_{\{p_1\}} = \arg_{\{p_1\}} \max \log_2 \det \left( I_{NR} + \frac{Ex}{\mathbf{Q}N_o} \mathbf{H}_{\{p_1\}} \mathbf{H}_{\{p_1\}}^H \right) \text{ (bps} \cdot \text{Hz}^{-1}). \tag{7}$$

After selecting the first antenna, the second antenna is selected to maximize the channel capacity as:

$$p_2^{subplot} = \arg_{\{p_2 \neq p_1\}} \max_{subplot} C_{\{p_1^{subplot}, p_2\}}. \tag{8}$$

This process continues until all the  $Q$  antennas are selected. The additional  $(n + 1)^{th}$  antenna maximizes the channel capacity as:

$$p_{n+1}^{subplot} = \arg \max_{H_{\{l\}}} \left[ \frac{\mathbf{Q}N_o}{Ex} I_{NR} + \dots \right. \tag{9}$$

$$l \notin \{p_1^{subplot}, \dots, p_2^{subplot}\}$$

$$\left. \dots \mathbf{H}_{\{p_1, \dots, p_n^{subplot}\}} \mathbf{H}_{\{p_1, \dots, p_n^{subplot}\}}^H \right]^{-1} \mathbf{H}_{\{l\}}^H.$$

On the other hand, the same process can be implemented by deleting the antenna in Descending Order of decreasing channel capacity. From the performance perspective, the selection method in descending order outperforms that in ascending order when  $1 < Q < N_T$ . This is due to the fact that the selection method in descending order considers all correlations between the column vectors of the original channel gain before choosing the first antenna to delete. However, the complexity of the selection method in decreasing order is higher than that in increasing order.

### 3.2. Correlation

The capacity increases linearly with  $\min(N_T, N_R)$  compared to a conventional single-input single-output (SISO) systems. This increase in capacity requires a scattering environment, such that the channel gains matrix between transmit and receive antennas has a full rank and independent entries; and moreover, perfect estimation of channel gains at the transmitter or/and receiver. Hence, the MIMO channel capacity heavily depends on the statistical properties and antenna element correlations of the channel. The effect of channel correlation on capacity also depends on what is known about the channel at the transmitter and receiver. For Rayleigh Fading Channels, the channel correlation always reduces capacity and therefore independent identically distributed (i.i.d. Rayleigh) channels yields the maximum ergodic capacity. Disregarding bandwidth, the ergodic capacity of MIMO system with  $N$  antennas at both ends of the link is expressed as [11], [16]:

$$C_{iid} = E_H \{ \log_2 \det \left( I_n + \frac{\rho}{N} \mathbf{H} \mathbf{H}^H \right) \}, \tag{10}$$

where  $I_n$  denotes the  $n \times n$  identity matrix,  $\rho$  is the average received SNR and  $\mathbf{H}$  is the normalized channel matrix. Strictly, Eq. (10) represents the true capacity only in i.i.d. scenarios. While, generally, the MIMO channel gains are not independent and identically distributed.

The channel correlation is closely related to the capacity of the MIMO channel as it provides a lower

bound on the capacity, since it signifies the mutual information with a non-optimal power allocation (equal allocation). The capacity of the MIMO channel is considered with the correlated channel gains between transmit and received antennas. At high SNR, the deterministic channel capacity is estimated as [6]:

$$C = \max_{Tr(R_{xx})=N} \log_2 \det(\mathbf{R}_{xx}) + \log_2 \det\left(\frac{Ex}{NN_o} \mathbf{H}_w \mathbf{H}_w^H\right). \tag{11}$$

The second term in Eq. (11) is constant while the first term is maximized when  $R_{xx} = I_N$ . The correlated channel model is considered as [11]:

$$\mathbf{H} = \mathbf{R}_t^{\frac{1}{2}} \mathbf{H}_w \mathbf{R}_r^{\frac{1}{2}}, \tag{12}$$

where  $R_t$  is the correlation matrix, providing correlations between the transmit antennas (column vectors of channel matrix  $\mathbf{H}$ ),  $R_r$  is the correlation matrix providing correlations between the receive antennas (row vectors of  $\mathbf{H}$ ), and  $\mathbf{H}_w$  denotes the i.i.d. channel gain matrix. When  $N_T = N_R = N$ ,  $R_r$  and  $R_t$  are of full rank, and SNR is high, MIMO channel equation is estimated as [6]:

$$C \approx \log_2 \det\left(\frac{Ex}{NTNo} \mathbf{H}_w \mathbf{H}_w^H\right) + \log_2(R_r) + \log_2(R_t). \tag{13}$$

It indicates that the MIMO channel capacity is reduced and the amounts of reduction in capacity due to the correlation between transmits and receive antennas is [6]:

$$\log_2 \det(R_r) + \log_2 \det(R_t). \tag{14}$$

The value in Eq. (14) is always negative since  $\log_2 \det(\mathbf{R}) \leq 0$  for any correlation matrix  $\mathbf{R}$ . Also from the mathematical point of view [6], it is apparent that:

$$\log_2 \det(\mathbf{R}) \leq 0. \tag{15}$$

And the equality holds when the correlation matrix is the Identity Matrix and thus, the quantities in Eq. (14) are negative.

## 4. Simulation Methodology and Environment

To study the effect of Antenna Selection on the performance of MIMO wireless systems, the capacity of

various MIMO systems in Nakagami-m and Rayleigh fading channels are examined and compared. This paper addresses transmission techniques that exploit the channel state information (CSI) on the transmitter. The CSI can be partially or completely known on the transmitter side. Exploitation of such channel information allows for increasing the channel capacity, improving the error performance, while reducing the hardware complexity. The antenna selection techniques that exploit CSI at the transmitter are considered, that is, Optimum and Suboptimal antenna selections techniques. The algorithms for selecting the optimal antennas are as follows:

Algorithm of Suboptimal antenna selection method in decreasing order:

- Initially all the antennas are considered, i.e.  $S_1 = \{1, 2, \dots, N_T\}$ .
- The antenna contributing least to the capacity is selected and then this selected antenna is deleted from the antenna index set.
- The remaining antenna set is updated to  $S_2$ .
- If  $|S_2| = N_T - 1 > Q$ , then another antenna that contributes least to the capacity is deleted.
- Again the remaining antenna set is updated.
- This process continues until all  $Q$  antennas are selected, i.e.  $|S_N| = Q$ .

Algorithm of Suboptimal antenna selection technique in increasing order:

- Initially all the antennas are considered.
- One antenna with the highest capacity is selected.
- Given first selected antenna, the second antenna is selected such that channel capacity is maximized.
- After  $n^{th}$  iteration, the capacity with additional antenna can be updated.
- The additional  $(n + 1)^{th}$  antenna is the one that maximizes the channel capacity.
- This process continues until all the  $Q$ 's are selected (i.e.  $n + 1 = Q$ ).

Besides, to study the Effect of Correlation, a MIMO system with the channel gains between the transmitter and receiver antennas are correlated. The Ergodic capacity is evaluated, which is attained as the mean value of capacity obtained from a number of independent channel realizations. The simulations are done here by considering the different channel equations for Nakagami-m and Rayleigh fading channel. The effect of correlation on the performance of MIMO wireless systems is evaluated as follows:

- The correlated MIMO channel model  $\mathbf{H}$  is considered.
- The  $N_T$  transmit and  $N_R$  receive antennas are initialized.
- The transmit correlation matrix  $R_t$  is initialized that provide correlation between the transmit antennas, means correlation between the column vectors of  $\mathbf{H}$ .
- The receive correlation matrix  $R_r$  is initialized that provide correlation between the receive antennas, means correlation between the row vectors of  $\mathbf{H}$ , such that diagonal entities of correlation matrix are constrained to unity.
- The channel is generated (Rayleigh or Nakagami-m).
- The i.i.d. and correlated channel capacity is evaluated by considering their basic capacity equations (Eq. (8) and Eq. (11)).

The performance of MIMO systems through fading channels, with antenna selection and correlation is governed by MIMO simulation environment defining a range of parameters. This paper is based on a simple channel model, assuming the channel to be a random matrix and is subjected to Nakagami and Rayleigh fading. The Rayleigh channel model represents the scattered signals that arrive at receiver via multiple paths (i.e. multipath propagation). The Nakagami-m channel model is another important channel used in wireless systems to model the statistical fading of multipath scenarios and matches the empirical data better than other channel models. The Nakagami-m parameter used for simulations is  $m = 4$ . The simulation environment parameters used are as tabulated below. Here SNR is the ratio of signal power to noise power (in decibels) and Iteration is the number of repetition of process.  $N_T$  and  $N_R$  are the number of antennas at transmitter and receiver side of the communication link. The  $Q$  or sel-ant are the best (optimal) antennas selected in the Antenna Selection Techniques. The simulation parameters used are shown in Tab. 1.

### 5. Results and Discussion

In this paper, the capacity bound dependencies are illustrated under various fading channel models, i.e., MIMO Rayleigh fading and Nakagami-m fading channels. The performance of MIMO systems with antenna selection at the transmitter side is illustrated in terms of capacity. It is reviewed that the channel capacity of the system not only depends on the number of transmit (and/or receiver) antennas chosen, but also on which transmit antennas are chosen.

Tab. 1: List of simulation parameters.

Simulation parameter		Antenna selection	Correlation
SNR	SNR [dB]	0–20 dB	0–20 dB
Iterations	Number of channel realizations	1000	1000
NT	Number of transmit antennas	4 or 8	4 or 8
NR	Number of receive antennas	4 or 8	4 or 8
Q or selant	Antennas selected	1, 2, 3, 4 or 1, 2, 3, . . . , 8	-
Channel	Fading channel used	Nakagami-m and Rayleigh fading channel	Nakagami-m and Rayleigh fading channel

Figure 2 shows the Channel capacity with optimal antenna selection for  $N_T = N_R = 4$  and the number of selected antennas  $Q = sel - ant = 1, 2, 3, 4$ . The channel capacity increases in proportion to the number of the selected antennas. For SNR less than 10 dB, the selection of three antennas in Rayleigh fading channel is enough to achieve the same channel capacity as achieved with all the four antennas. Also it can be seen that at SNR of 14 dB (for example), the capacity of  $sel - ant = 2$  in Rayleigh channel is 7  $\text{bps}\cdot\text{Hz}^{-1}$  while in Nakagami-m channel it is 21.5  $\text{bps}\cdot\text{Hz}^{-1}$ . Hence, the capacity with Nakagami-m is greater than Rayleigh distribution, for the same value of SNR.

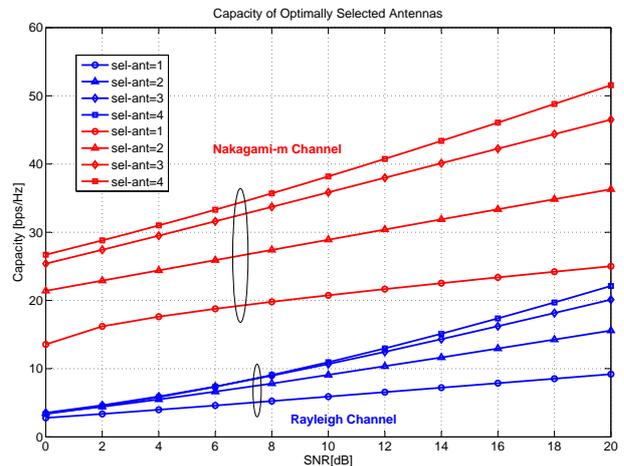


Fig. 2: Channel capacity with optimal antenna selection:  $N_T = N_R = 4$  and  $sel - ant = 1, 2, 3, 4$ .

Figure 3 shows the Channel capacity with optimal antenna selection for  $N_T = N_R = 8$  and the number of selected antennas,  $sel - ant = 1, 2, 3, 4, 5, 6, 7, 8$ . It is clear that the channel capacity increases with the number of selected antennas and is further increased in Nakagami-m channel than that of Rayleigh fading channel; for example, at SNR of 14 dB, the capacity of

14  $\text{bps}\cdot\text{Hz}^{-1}$  is achieved with  $\text{sel-ant} = 2$  in Rayleigh fading channel while in Nakagami-m fading channel, the capacity of 36  $\text{bps}\cdot\text{Hz}^{-1}$  is achieved with  $\text{sel-ant} = 2$  at same SNR of 14 dB.

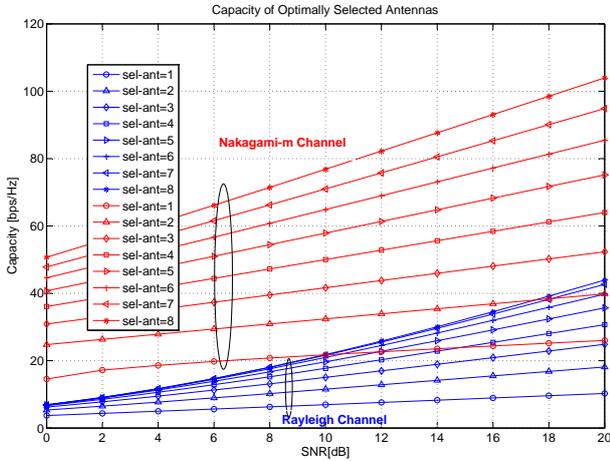


Fig. 3: Channel capacity with optimal antenna selection:  $N_T = N_R = 8$  and  $\text{sel-ant} = 1, 2, 3, 4, 5, 6, 7, 8$ .

Figure 4 shows the channel capacity with the selection method in ascending order for various selected antennas with  $N_T = 4$  and  $N_R = 4$ . At SNR of 14 dB, the capacity of suboptimal antenna selection with 1, 2, 3 and 4 selected antennas in increasing order is respectively 7  $\text{bps}\cdot\text{Hz}^{-1}$ , 11.5  $\text{bps}\cdot\text{Hz}^{-1}$ , 14.5  $\text{bps}\cdot\text{Hz}^{-1}$  and 15.2  $\text{bps}\cdot\text{Hz}^{-1}$  in Rayleigh channel while in Nakagami-m channel, the capacity of suboptimal antenna selection in increasing order at SNR of 14 dB is 24  $\text{bps}\cdot\text{Hz}^{-1}$ , 34.5  $\text{bps}\cdot\text{Hz}^{-1}$ , 43  $\text{bps}\cdot\text{Hz}^{-1}$  and 48  $\text{bps}\cdot\text{Hz}^{-1}$ , respectively, which indicates the Nakagami-m channel provides better performance. Also, when the curves in Fig. 4 are compared with those in Fig. 2, it can be seen that the suboptimal antenna selection method for  $Q$  or  $\text{sel-ant} = 1$ , in Rayleigh fading achieves the same channel capacity as the optimal antenna selection method for  $\text{sel-ant} = 1$ . However, the capacity is increased for other selected antennas in Rayleigh fading channel; and for all selected antennas in Nakagami-m fading channel.

Figure 5 shows the channel capacity with the selection method in increasing order of selected antennas with  $N_T = 8$  and  $N_R = 8$ . The channel capacity is increased with the number of the selected antennas. Moreover, at SNR of 14 dB, the capacity of MIMO with a suboptimally selected antenna,  $Q$  or  $\text{sel-ant} = 4$  and 6 in Nakagami-m channel is improved by 36  $\text{bps}\cdot\text{Hz}^{-1}$  and 51  $\text{bps}\cdot\text{Hz}^{-1}$  respectively; than that of  $\text{sel-ant} = 4$  and 6 in Rayleigh fading channel.

Figure 6 shows the channel capacity with the selection method in descending order for various numbers of selected antennas with  $N_T = 4$  and  $N_R = 4$ . The capacity for the antenna selection is lower with Rayleigh distribution than with Nakagami-m channel, at the

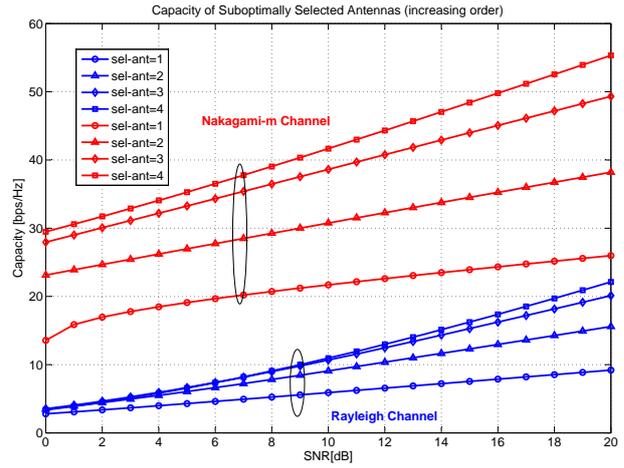


Fig. 4: Channel capacities for antenna selection method in increasing order with  $N_T = N_R = 4$ .

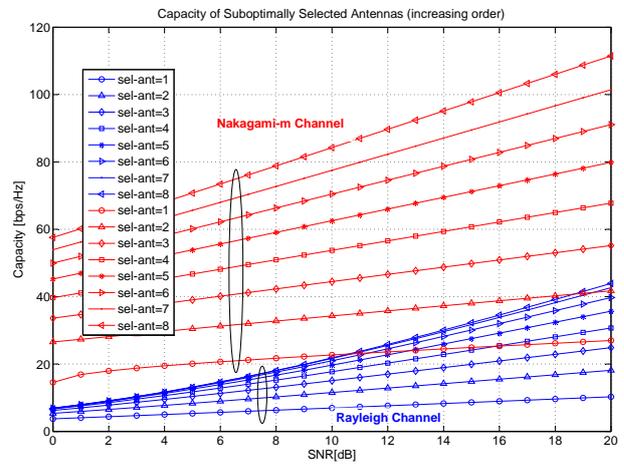


Fig. 5: Channel capacities for antenna selection method in increasing order with  $N_T = N_R = 8$ .

same SNR; for example, at SNR of 14 dB, the capacity in Rayleigh fading channel with  $\text{sel-ant} = 1, 2, 3, 4$  is respectively 7.5  $\text{bps}\cdot\text{Hz}^{-1}$ , 12  $\text{bps}\cdot\text{Hz}^{-1}$ , 16  $\text{bps}\cdot\text{Hz}^{-1}$  and 18  $\text{bps}\cdot\text{Hz}^{-1}$ ; while in Nakagami-m fading channel, the capacity is 24  $\text{bps}\cdot\text{Hz}^{-1}$ , 35  $\text{bps}\cdot\text{Hz}^{-1}$ , 46  $\text{bps}\cdot\text{Hz}^{-1}$  and 53  $\text{bps}\cdot\text{Hz}^{-1}$ , respectively.

Figure 7 shows the channel capacity with the selection method in decreasing order for various numbers of selected antennas with  $N_T = 8$  and  $N_R = 8$ . At SNR of 14 dB, the capacity of MIMO with suboptimally decreasingly selected antenna,  $Q$  or  $\text{sel-ant} = 4$  and 6 in Nakagami-m channel is improved by 36  $\text{bps}\cdot\text{Hz}^{-1}$  and 52  $\text{bps}\cdot\text{Hz}^{-1}$ , respectively, than that of  $\text{sel-ant} = 4$  and 6 in Rayleigh fading channel; which indicates that the capacity is improved in Nakagami-m channel than that of Rayleigh channel, at same SNR. Moreover, from Fig. 6 and Fig. 7, it can be seen that the capacity is improved in suboptimal antenna selection in decreasing order than that of optimal antenna selection and suboptimal antenna selection in increasing order.

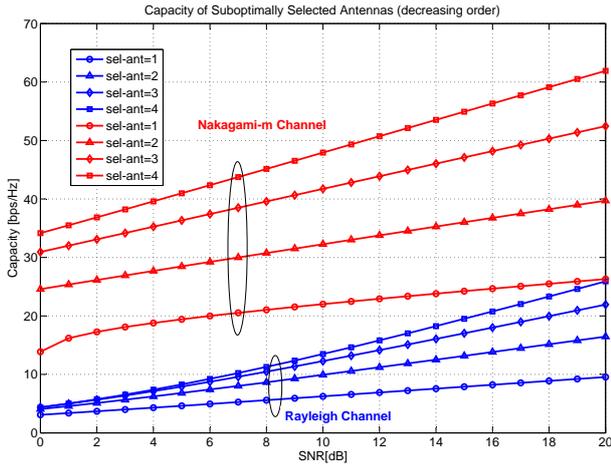


Fig. 6: Channel capacities for antenna selection method in decreasing order with  $N_T = N_R = 4$ .

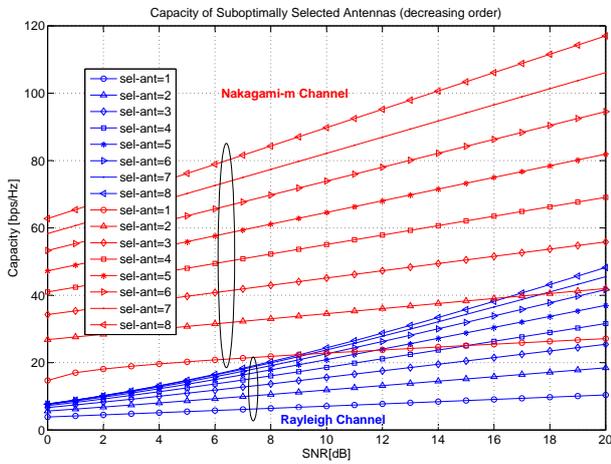


Fig. 7: Channel capacities for antenna selection method in decreasing order with  $N_T = N_R = 8$ .

The acquired simulation results are now tabulated, to compare various antenna selection techniques and fading channels used. The number of selected antennas is set by varying the parameter  $Q$  or  $sel - ant$ . The channel capacity is increased in proportion to the number of the selected antennas and further increased with the SNR. Moreover, the capacity is improved in Nakagami- $m$  channel than that of Rayleigh channel at same value of SNR. From the results obtained, it can be seen that the same capacity is achieved  $sel - ant = 1$  for optimal antenna selection and suboptimal antenna selection in increasing order, under Rayleigh fading channel. Thus to achieve the same capacity, lesser number of selected antennas are required in Rayleigh fading channel than that of Nakagami- $m$  channel. For example, the capacity of  $20 \text{ bps}\cdot\text{Hz}^{-1}$  is achieved by selecting three antennas ( $sel - ant = 3$ ) at SNR of 20 dB, for Optimal Selection method; whereas only one antenna selection ( $sel - ant = 1$ ) at SNR of 10 dB is enough to achieve almost the same capacity of  $20.5 \text{ bps}\cdot\text{Hz}^{-1}$  in Nakagami- $m$  channel. Similarly, for Suboptimal An-

tenna Selection, the capacity of  $27 \text{ bps}\cdot\text{Hz}^{-1}$  is achieved with  $sel - ant = 4$  at SNR of 20 dB in Rayleigh channel; whereas the same capacity can be achieved with  $sel - ant = 1$  at SNR of 20 dB in Nakagami- $m$  channel. Hence, the capacities of antenna selection methods are improved in Nakagami- $m$  channel than that of Rayleigh channel. Moreover, the suboptimal antenna selection in decreasing order provides the maximum capacity over techniques.

To evaluate the effect of correlation on the capacity of MIMO fading channels, the capacity of the typical i.i.d. MIMO channel is compared with the capacity of a correlated MIMO channel. Figure 8 plots the  $4 \times 4$  MIMO ergodic channel capacity computed when there exists a correlation between the transmit and receive antennas. The result shows that the capacity of MIMO channel with correlation is reduced than that of i.i.d. channel. At SNR of 18 dB, the capacity of  $3.3 \text{ bps}\cdot\text{Hz}^{-1}$  is lost due to the channel correlation in Rayleigh fading channel and  $2.2 \text{ bps}\cdot\text{Hz}^{-1}$  is lost in Nakagami- $m$  fading channel due to correlation. It can also be seen that for SNR below 2 dB in Nakagami- $m$  channel, the correlated channel has a larger capacity than the i.i.d. channel; however, generally it is believed that the rich scattering environment is required for the optimal use of multiple antennas. This crossover is due to the reason that multiplexing gain offered by i.i.d. channel manifests itself at sufficiently high SNR only. Afterwards, for SNR below 6 dB, the channel capacity is found to be nearly the same for the i.i.d. and correlated MIMO, in Nakagami- $m$  channel.

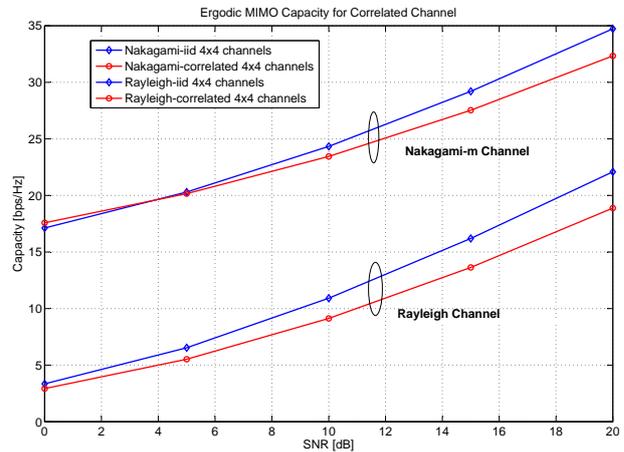


Fig. 8: Capacity reduction due to the channel correlation  $N_T = N_R = 4$ .

Figure 9 plots the MIMO ergodic channel capacity when there exists a correlation between the transmit and receive antennas where  $N_T = N_R = 4$ . The result shows that the capacity of  $9.9 \text{ bps}\cdot\text{Hz}^{-1}$  is lost due to the channel correlation in Rayleigh fading channel where as  $7.2 \text{ bps}\cdot\text{Hz}^{-1}$  is lost in Nakagami- $m$  fading channel, at SNR of 18 dB. Means the channel correla-

Tab. 2: Capacity of MIMO channels with Antenna Selection.

Antenna selection method	Num. of selected antennas where $N_T = N_R = 4$	Rayleigh channel		Nakagami-m channel	
		SNR = 10 dB	SNR = 20 dB	SNR = 10 dB	SNR = 20 dB
Optimal antenna selection	sel-ant=1	6 bps·Hz <sup>-1</sup>	9 bps·Hz <sup>-1</sup>	20.5 bps·Hz <sup>-1</sup>	25 bps·Hz <sup>-1</sup>
	sel-ant=2	9 bps·Hz <sup>-1</sup>	15 bps·Hz <sup>-1</sup>	28.5 bps·Hz <sup>-1</sup>	36 bps·Hz <sup>-1</sup>
	sel-ant=3	10.5 bps·Hz <sup>-1</sup>	20 bps·Hz <sup>-1</sup>	36 bps·Hz <sup>-1</sup>	46 bps·Hz <sup>-1</sup>
	sel-ant=4	10.5 bps·Hz <sup>-1</sup>	22 bps·Hz <sup>-1</sup>	38 bps·Hz <sup>-1</sup>	52 bps·Hz <sup>-1</sup>
Suboptimal antenna selection	sel-ant=1	6 bps·Hz <sup>-1</sup>	9 bps·Hz <sup>-1</sup>	21.5 bps·Hz <sup>-1</sup>	26.5 bps·Hz <sup>-1</sup>
	sel-ant=2	9.5 bps·Hz <sup>-1</sup>	15.5 bps·Hz <sup>-1</sup>	30.5 bps·Hz <sup>-1</sup>	38.5 bps·Hz <sup>-1</sup>
	sel-ant=3	11 bps·Hz <sup>-1</sup>	20 bps·Hz <sup>-1</sup>	38 bps·Hz <sup>-1</sup>	49.5 bps·Hz <sup>-1</sup>
	sel-ant=4	11 bps·Hz <sup>-1</sup>	22.5 bps·Hz <sup>-1</sup>	42 bps·Hz <sup>-1</sup>	56 bps·Hz <sup>-1</sup>
Subopt. ant. selection in decreasing order	sel-ant=1	6 bps·Hz <sup>-1</sup>	9.8 bps·Hz <sup>-1</sup>	21 bps·Hz <sup>-1</sup>	27 bps·Hz <sup>-1</sup>
	sel-ant=2	10 bps·Hz <sup>-1</sup>	16.5 bps·Hz <sup>-1</sup>	32 bps·Hz <sup>-1</sup>	39.8 bps·Hz <sup>-1</sup>
	sel-ant=3	12 bps·Hz <sup>-1</sup>	22 bps·Hz <sup>-1</sup>	42 bps·Hz <sup>-1</sup>	52.5 bps·Hz <sup>-1</sup>
	sel-ant=4	13 bps·Hz <sup>-1</sup>	27 bps·Hz <sup>-1</sup>	48 bps·Hz <sup>-1</sup>	62.5 bps·Hz <sup>-1</sup>

tion reduces capacity of MIMO channel and this reduction further increased with SNR. However, below 1 dB, the capacity in Nakagami-m channel is larger for i.i.d. channel than that for correlated channel, and is nearly the same below 3 dB; since the multiplexing gain offered by i.i.d. channel in Nakagami-m manifests itself at sufficiently high SNR.

smaller for Nakagami-m channel than that for Rayleigh channel.

Tab. 3: Capacity of MIMO channels with and without correlation.

MIMO channels	Rayleigh channel		Nakagami-m channel	
	SNR= 10 dB	SNR= 20 dB	SNR= 10 dB	SNR= 20 dB
i.i.d. 4 × 4	11 bps·Hz <sup>-1</sup>	22 bps·Hz <sup>-1</sup>	26.2 bps·Hz <sup>-1</sup>	37.5 bps·Hz <sup>-1</sup>
Correlated 4 × 4	9 bps·Hz <sup>-1</sup>	18.5 bps·Hz <sup>-1</sup>	25.2 bps·Hz <sup>-1</sup>	35 bps·Hz <sup>-1</sup>
i.i.d. 8 × 8	22 bps·Hz <sup>-1</sup>	44 bps·Hz <sup>-1</sup>	33 bps·Hz <sup>-1</sup>	53 bps·Hz <sup>-1</sup>
Correlated 8 × 8	16 bps·Hz <sup>-1</sup>	34 bps·Hz <sup>-1</sup>	29 bps·Hz <sup>-1</sup>	45 bps·Hz <sup>-1</sup>

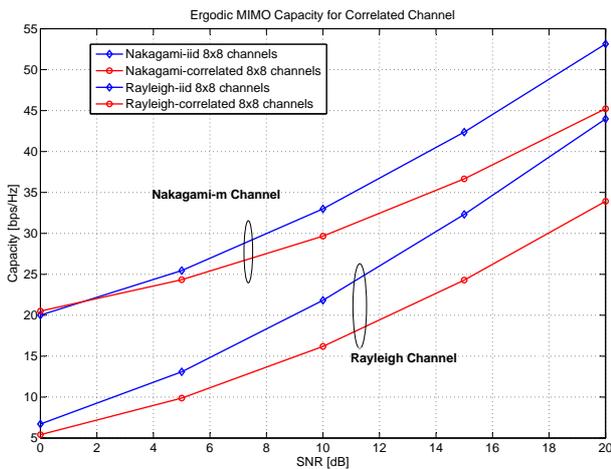


Fig. 9: Capacity reduction due to the channel correlation  $N_T = N_R = 8$ .

The results from Fig. 8 and Fig. 9 are tabulated in Tab. 3, which shows the resulting average  $4 \times 4$  and  $8 \times 8$  MIMO capacity of an i.i.d. and a correlated channel as a function of the receive SNR. Evidently, the correlated channel exhibits lower ergodic capacity. The difference for  $8 \times 8$  MIMO systems is more than  $4 \times 4$  systems; and it is further increased with the SNR. It reviewed the general statement that i.i.d. channel gives maximum ergodic capacity. At SNR of 20 dB, the capacity of  $3.5 \text{ bps}\cdot\text{Hz}^{-1}$  is lost in Rayleigh channel and  $1.3 \text{ bps}\cdot\text{Hz}^{-1}$  is lost in Nakagami-m channel, for  $4 \times 4$  MIMO systems. It indicates the reduction in channel capacity due to correlation; however this reduction is

## 6. Conclusion

MIMO wireless communications have a great potential for achieving high data rates, by using multiple antennas at the transmitter and receiver. In this paper, the performance of MIMO wireless systems over Nakagami-m and Rayleigh fading channels with Antenna Selection and Correlation is analyzed using different antenna configurations. The Antenna Selection Techniques are used to employ smaller number of RF modules than the number of transmit/receive antennas by selecting the best  $Q$  out of  $N$  available antennas and thus reducing RF chains to  $Q$  instead of  $N$ . The Optimum and Suboptimal antenna selection techniques are described. The antennas in suboptimal method are selected in ascending order of increasing capacity or descending order of decreasing capacity; which further reduces the system complexity over optimal antenna selection. However, the same capacity could be achieved with both methods.

Besides, the effect of correlation on the performance of MIMO fading channels is evaluated by analyzing the

ergodic MIMO channel capacity when there exists a correlation between the transmit and receive antennas. It is verified that the correlation reduces the MIMO channel capacity. This reduction in capacity is further increased with the SNR and number of antennas at the transmitter and receiver. The obtained results give an inspection to the influence of fading channels over the capacity bounds of MIMO wireless systems.

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