Comparison of Numerical Modelling of Degradation Mechanisms in Single Mode Optical Fibre Using MATLAB and VPIphotonics

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Abstract. *Mathematical models for description of physical phenomena often use the statistical description of the individual phenomena and solve those using suitable methods. If we want to develop numerical model of optical communication system based on transmission through single mode optical fibres, we need to consider whole series of phenomena that affect various parts of the system. In the single-mode optical fibre we often encounter influence of chromatic dispersion and nonlinear Kerr effects. By observing various different degradation mechanisms, every numerical model should have its own limits, which fulfil more detailed specification. It is inevitable to consider them in evaluation. In this paper, we focus on numerical modelling of degradation mechanisms in single-mode optical fibre. Numerical solution of non-linear Schroedinger equation is performed by finite difference method applied in MATLAB environment and split-step Fourier method, which is implemented by VPIphotonics software.*

Keywords

Finite difference method, nonlinear Schroedinger equation, single mode optical fibre, split-step Fourier method.

1. Introduction

Development of technologies and the related growth of requirements on transmission capacity of optical communication systems leads to development and implementation of new technologies. Numerical modelling becomes inevitable for research and design of new devices and systems [1], [2].

Nowadays for efficient transmission of big amount of data, single-mode optical fibres (SMF) are used, which have the best premise to fulfil the transmission requirements of broad-band services utilising wavelength division and polarisation division multiplexing. The main problems related to long-distance high-speed optical communication systems are caused by linear and nonlinear phenomena. Based on physical descriptions and by implementing various different numerical methods for solving them, we can nowadays express transmission and signal characteristics of almost all optical components [3].

There are several well-established numerical methods for numerical modelling of transmission signal via SMF fibre [1], [3], [4], [5]. Numerical methods that are used very often are pseudo-spectral time differentials and elements. From implementation aspects, split-step Fourier method (SSFM) and finite difference method (FDM) are most suitable. These methods are implemented by various algorithms depending on their computational efficiency and accuracy [3], [6].

2. Theory

Propagation of the optical pulse in SMF fibre usually described by the nonlinear Schroedinger differential equation (NLSE) [1], [3].

Using NLSE in Eq. (1), we can describe propagation of modulated optical signal in SMF by using complex envelope $A(z,t)$, which includes all degradation mechanisms. NLSE describes signal propagation in nonlinear dispersive fibre, while it can take various shapes, which depends on examination of individual phenomena and established simplifications. This form of Schroedinger equation includes attenuation, chromatic dispersion and self-phase modulation. NLSE can be for SMF expressed as follows:

$$\frac{\partial A(z,t)}{\partial z} = j\gamma |A(z,t)|^2 A(z,t) - \\ - j\frac{\beta_2}{2}\frac{\partial^2 A(z,t)}{\partial t^2} - \frac{\alpha}{2}A(z,t), \quad (1)$$

where parameter γ is non-linear coefficient ($W^{-1}\cdot km^{-1}$), β_2 ($ps^2\cdot km^{-1}$) characterizes group velocity dispersion (GVD) and parameter α represents fibre loss [3], [7].

The loss of power depends on the wavelength of the light and on the propagating material. Modern optical fibres have loss approximately 0.2 $dB\cdot km^{-1}$ in the proximity of wavelength $\lambda_0 = 1550$ nm.

The dispersion of fibre influences propagation of optical signal in the time domain, i.e. transmission properties. Each spectral component of transmitting signal is therefore propagated through fibre with different group velocity.

Nonlinear coefficient of refraction index is a parameter dependent on wavelength [8]. Its value for silicon glass is approximately $2.6 \cdot 10^{-20}$ ($m^2\cdot W^{-1}$) and depends on dopant concentration inside the core. If the refraction index is dependent on the light intensity entering this environment, we speak about Kerr phenomena. This change of refraction index, which is different in different parts of pulse significantly influences changes in shape and pulse polarisation during propagation [3], [7].

2.1. Gaussian Shape of Optical Pulse

In the optical communication systems, the transmitting signal can have various shape. For approximation is done very often using Gaussian function of pulse shape:

$$A(z,t) = \sqrt{P_{in}} e^{\left(-\frac{1+jC}{2}\right)\left(\frac{t}{T_0}\right)^{2m}}, \quad (2)$$

where P_{in} refers to input power, C represents the initial chirp (optical pulses generated by directly modulated lasers and by certain types of externally modulated lasers show frequency chirp, which represents the change of optical carrying frequency of given pulse because of the modulation), T_0 represents the initial width of optical pulse during decrease to $1/e$ from maximal amplitude and m is parameter of optical pulse shape (for Gauss pulse $m = 1$, $m = 3$, for so called Super-Gauss pulse) [3].

3. Numerical Methods for Modelling SMF Fibre

SSFM method is commonly used for analysis of nonlinear effects in optical fibre. However, its use is time consuming in scenarios like solving coupled nonlinear Schrodinger equations for systems with wavelength division multiplex. FDM method has several schemas for solving such equations. They have different computational complexity and solution accuracy. We differentiate implicit, explicit and Crank-Nicholson scheme. These numerical methods are a suitable tool for solving NLSE, which in general does not have any analytical solution [3], [6], [9], [10], [11].

3.1. SSFM Method

Split-step Fourier method is a numerical pseudo-spectral method, which name is derived from the method of NLSE result computation. The computation is performed in small steps and linear and nonlinear part are solved separately. The inevitability of this method is utilisation of algorithms FT and IFT (Fourier and inverse Fourier transformation), because solving of linear phenomena (dispersion and loss) is realised in spectral domain, while nonlinear phenomena are solved in time domain. Using fast Fourier transformation algorithm speeds up the computation [3], [7], [11].

For understanding the method, we state simplified form of the algorithm from Eq. (1):

$$\frac{\partial A}{\partial z} = (\hat{D} + \hat{N})A, \quad (3)$$

where $\hat{D}$ considers linear effects and $\hat{N}$ considers nonlinear Kerr effects, which we can express by following equations [7], [11]:

$$\hat{D}A = -j\beta_2\frac{\partial^2 A}{\partial \tau^2} - \frac{\alpha}{2}A, \quad (4)$$

$$\hat{N}A = iy|A|^2 A, \quad (5)$$

Since the mechanisms considered in numerical SSFM method are solved as individually acting effects, we can express the resulting complex envelope $A(z+h,\tau)$, which integrates individual analytical solutions:

$$A(z+h,\tau) = A(z,\tau)e^{h\hat{D}}e^{h\hat{N}}, \quad (6)$$

where h represents the computational step size.

3.2. FDM Method

Finite difference method is a numerical method for differential equations approximation. One of the biggest advantages of FDM method is that whole computation is performed only in time domain. It uses approximation of function derivation $f'(x)$ in given point using function values in neighbouring points. Explicit FTCS scheme has the following form [6], [10]:

$$\frac{f_i^{n+1} - f_i^n}{\triangle t} = D\left[\frac{f_i^n + 1 + f_i^n - 1 - 2f_i^n}{(\triangle x)^2}\right]. \tag{7}$$

Using rules of FTCS scheme, the solution of Schroedinger Eq. (1) is as follows:

$$\begin{aligned} A(z+hz,t) = \\ A(z,t)\left[1 - 2R + j\gamma|a(z,t)|^2 hz - \frac{\infty}{2}hz\right] + \\ +R[A(z,t+ht) + A(z,t-ht)], \end{aligned} \tag{8}$$

$$R = -j\frac{\beta_2}{2}\frac{hz}{ht^2}, \tag{9}$$

where hz represents step size of the computation in space and ht represents the step size of the computation in time.

4. Experiments

We can efficiently perform simulations of individual physical effects in the field of optical communication systems using MATLAB. We applied FTCS scheme was applied in this environment, where we transmitted optical pulse of Gaussian form of wavelength $\lambda_0 = 1550$ nm. We used SMF fibre with dispersion parameter $D = 16$ ps$\cdot$(nm$\cdot$km)$^{-1}$ and nonlinear refraction index $n_2 = 2.475 \cdot 10^{-20}$ m$^2\cdot$W^{-1} with zero attenuation [3]. Input power bounded into optical fibre was $P_{in} = 1$ mW and length of optical fibre was in range $L = 10-100$ km. Input and output signal are displayed in Fig. 1 (fibre of length $L = 50$ km was used). Very small dispersion of pulse is because of bandwidth of optical signal is considered zero and duration of Gaussian pulse is very long in comparison with the fibre dispersion.

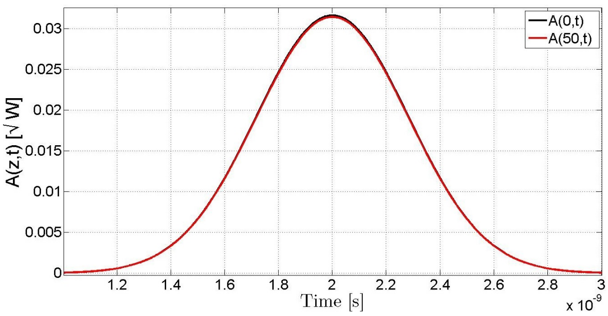

Fig. 1: Propagation of optical pulse in SMF fibre of length $L = 50$ km using FTCS scheme applied in MATLAB environment.

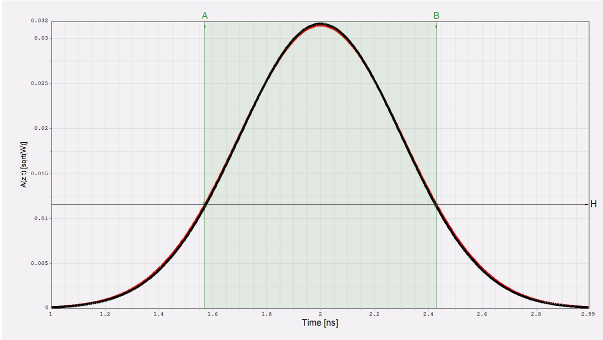

Fig. 2: Propagation of optical pulse in SMF fibre of length $L = 50$ km using SSFM scheme applied in VPIphotonics.

Virtual Photonics Incorporated (VPIphotonics) environment provides tools for optical network simulation, systems and devices. For simulation of optical pulse propagation through SMF fibre, it uses SSFM method. In VPIphotonics, we performed the simulation in the same conditions as in the previous case. Transmission of optical signal with optical fibre length 50 km is depicted in Fig. 2.

Figure 3 and Fig. 4 displays maximal values of output signal amplitudes and full width at half maximum FWHM in dependency of optical fibre length for both methods used.

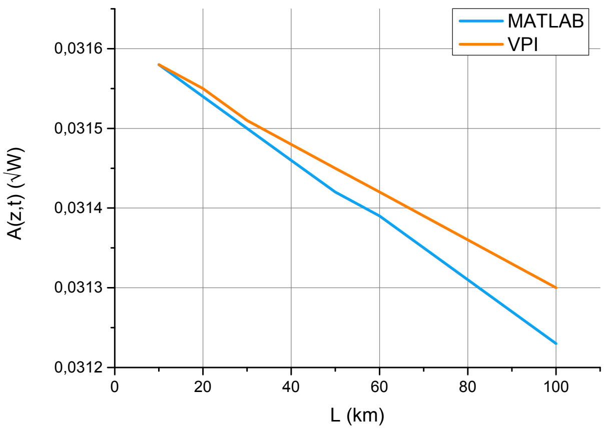

Fig. 3: Maximal values of output signal amplitudes in dependency of optical fibre length in both MATLAB and VPIphotonics environment.

Based on Fig. 3 we can conclude that mean relative variance between FDM method applied in MATLAB environment and SSFM method applied in VPIphotonics is $3.2 \cdot 10^{-5}$ %. By comparison of FWHM values in graph on Fig. 4, we can conclude that the accuracy of simulation results can be obtained with mean relative variance 0.0516 %, meaning that the both models have comparable results.

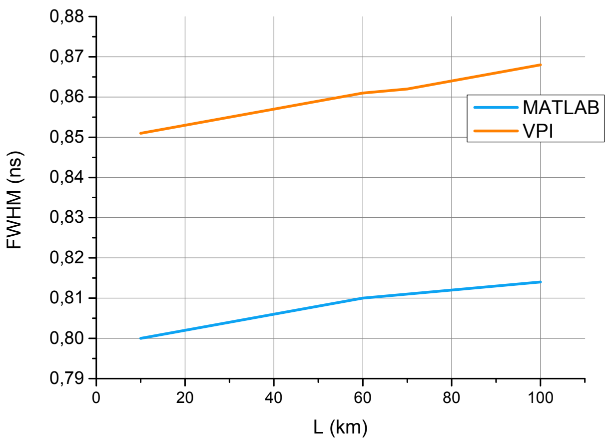

Fig. 4: FWHM in dependency of optical fibre length in MATLAB and VPIphotonics.

5. Conclusion

In this paper, we present results of application of FDM method and SSFM method for numerical modelling of degradation mechanisms in SMF fibre. Analysing the results obtained by numerical model realised in MATLAB environment using FDM method FTCS scheme and model implemented in VPIphotonics, which uses SSFM method. We conclude that measurement of FWHM of optical pulse propagation can be performed with mean relative variance approximately 0.05 %. Model that we created in MATLAB has comparable results with a model in the VPIphotonics. These models may be used for further study of optical pulse propagation through SMF fibre, i.e. signals in multi-channelled optical systems.

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Jana SAJGALIKOVA was born in Brezno, Slovakia in the 1989. In 2013 she finished M.Sc. at University of Zilina, Faculty of Electrical Engineering, Department of Telecommunications and Multimedia. Currently she studies doctor degree. Her research is focused on numerical modelling of selected parts of optical communication systems and their applications in high-speed optical networks.

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