# ANALYTICAL METHOD OF CALCULATION OF THE CURRENT AND TORQUE A RELUCTANCE STEPPER MOTOR VIA FOURIER SERIES

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**Summary:** Reluctance stepper motors are becoming to be very attractive transducer to conversion of electric signal to the mechanical position. Due to its simple construction is reluctance machine considered a very reliable machine which not requiring any maintenance. Present paper proposes a mathematical method of an analytical calculus of a phase current and electromagnetic torque of the motor via Fourier series. Saturation effect and winding reluctance are neglected.

## 1. INTRODUCTION

The reluctance stepper motor is an electromagnetic incremental actuator that can convert digital pulse inputs to analog output shaft motion. It is therefore used in digital control systems. A train of pulses is made to turn the shaft of the motor by steps. Neither a position sensor nor a feedback system is normally required for the reluctance stepper motor to make the output response follow to input command. Typical applications of the reluctance stepper motors requiring incremental motion are printers, desk drives, clocks and robotics.

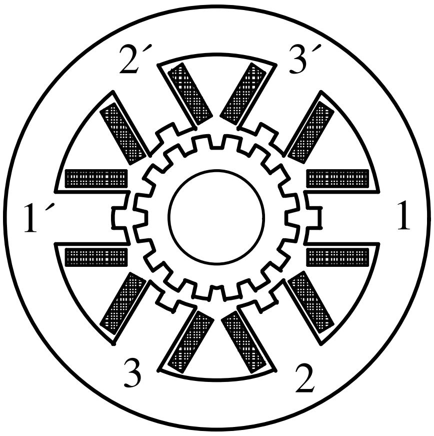


*Fig. 1. Reluctance stepper motor*

Due to its simple construction is reluctance machine considered a very reliable machine which not requiring any maintenance. Typical resolution of commercially available reluctance stepper motor ranges from several steps per revolution to as many as 400 steps per revolution.

A variable reluctance stepper motor can by of single stack type or the multi stack type constructed. A basic circuit configuration of the three phase single stack reluctance stepper motor is shown in Figure 1.

When the stator phases are excited with dc current in proper sequence, the resultant air gap field step around and the rotor follows the axis of the air gap field by virtue of reluctance torque. The reluctance torque is generated because of the tendency of the ferromagnetic rotor to align itself along the direction of the resultant magnetic field.

## 2. MATHEMATICAL MODEL OF THE MOTOR

The analysis of the reluctance stepper motor will be started from the electric model shown on the Figure 2. The mutual inductance between the phases is negligible, so it is sufficient to consider the only one phase of the motor. This one phase equivalent circuit comprises ohmic resistance of the coil winding and induced voltage caused by a change of stator inductance.

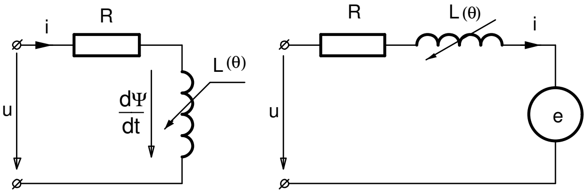


*Fig. 2. Equivalent circuit o reluctance motor*

For the one phase equivalent circuit a voltage equation is valid:

$$u = Ri + \frac{d\psi}{dt} \tag{1}$$

Where $\psi$ is total magnetic flux of the stator coil.

Generally a magnetization curve is expressed by:

$$\psi = L(\theta, i).i \tag{2}$$

Magnetic flux of the machine depends on the mutual position of the stator pole and the rotor tooth. In zone of the saturation it depends on the excitation current too.

The period of $L(\theta)$ is $\dfrac{2\pi}{N_r}$;

Where: $N_r$ is number of rotor teeth.

Define the electrical angle of rotor position: $\theta = N_r \theta_m$.

Where: $\theta_m$ is mechanical angle of rotor position;

Subsequently electrical angular velocity is given by: $\omega = N_r \omega_m$;

Where: $\omega_m$ is mechanical angular velocity;

Suppose saturation effect is neglected (motor is not saturated), so the voltage equation can be write:

$$u = Ri + L(\theta)\frac{di}{dt} + i\frac{dL(\theta)}{dt} \tag{3}$$

Where: $i\dfrac{dL(\theta)}{dt} = e$ is the induced electromotive force.

At steady state, the time can be replaced by an angle of the rotor position. Introduce $dt = \dfrac{d\theta}{\omega}$ the voltage equation take a form:

$$u = Ri + L(\theta)\omega\frac{di}{d\theta} + i\omega\frac{dL(\theta)}{d\theta} \tag{4}$$

Magnetic coenergy of a magnetic circuit is defined by:

$$W'_{em} = \int_0^{i_0} \psi di \tag{5}$$

So the instantaneous torque is given by:

$$m = \frac{\partial W'_{em}}{\partial \theta} \tag{6}$$

For not saturable magnetic circuit the instantaneous electromagnetic phase torque of the machine is given by:

$$m = \frac{1}{2} i^2 \frac{dL(\theta)}{d\theta} \tag{7}$$

The instantaneous torque of the machine is independent of current polarity and it is proportional to the inductance derivative.

## 3. PHASE INDUCTANCES

The waveform of the phase inductance for non-saturable motor has a form given on the Figure 3.

The waveform of the inductance can be expressed as a Fourier series:

$$L(\theta) = L_0 + \sum_{k=1}^{\infty} L_k \cos(k\theta) \tag{8}$$

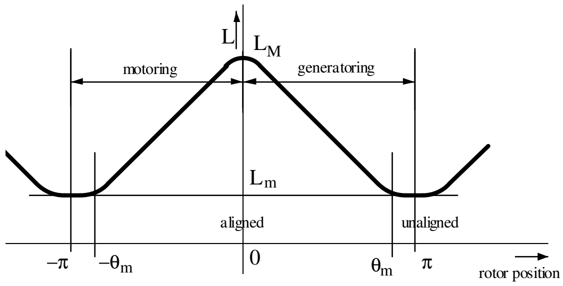


*Fig. 3. Course of the phase inductance versus rotor position*

Where: $L_0, L_k$ are the Fourier coefficients:

$$L_0 = L_m + \frac{L_M - L_m}{2\pi}\theta_m$$

$$L_k = \frac{2(L_M - L_m)}{k^2 \pi \theta_m}(1 - \cos k\theta_m)$$

Where: $L_M$ is maximal inductance in aligned rotor position;

$L_m$ is minimal inductance in unaligned rotor position;

$\theta_m$ is position of approaching the stator poles and rotor teeth;

For practical calculus it is sufficient to consider only five or six first members of the series.

## 4. SUPPLY CIRCUITS

The flux in reluctance motor is not constant, but it must be established from zero every stroke. In motoring operation the build-up is timed to coincide with the period when rotor poles are approaching the stator poles of the phase.

Assuming each phase is supplied by a circuit of the form given on the Figure 4.

Both transistors $T_1, T_2$ are switched at constant frequency and constant angle conductance. When the transistors are open, the voltage impressed on the stator winding is DC supply voltage. Consequently the supply current increases to its maximal value.

When the transistors are closed the stator winding is equal to the negation of the DC supply voltage, because current flow in the free wheeling diodes.

The waveform of the supply voltage can be expressed as a Fourier series of the form:

$$u = U\left\{\frac{a_0}{2} + \sum_{k=1}^{\infty}\left[a_k \cos(k\theta) + b_k \sin(k\theta)\right]\right\} \tag{9}$$

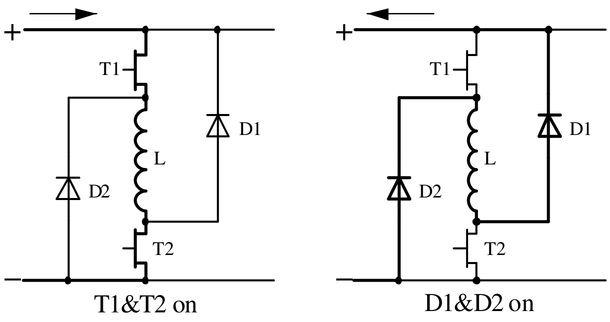


*Fig. 4. The supply circuit*

Where: $a_0, a_k, b_k$ are the calculated Fourier coefficients:

$$a_0 = \frac{1}{\pi}(\beta - \gamma)$$

$$a_k = \frac{1}{k\pi}\left[2\sin(\alpha+\beta)k - \sin\alpha k - \sin(\alpha+\beta+\gamma)k\right]$$

$$b_k = \frac{1}{k\pi}\left[\cos\alpha k - 2\cos(\alpha+\beta)k + \cos(\alpha+\beta+\gamma)k\right]$$

For: $k \neq 0$.

Where: $\alpha$ is turn on angle of the transistor;
$\beta$ is conductance angle of the transistors;
$\gamma$ is conductance angle of the diodes;

## 5. CURRENT AND TORQE CALCULATION

The waveform of the supply current is described by the differential voltage equation (4). After introducing (6) and (7) it takes a form:

$$U\left\{\frac{a_0}{2} + \sum_{k=1}^{\infty}\left[a_k \cos(k\theta) + b_k \sin(k\theta)\right]\right\} =$$

$$R.i + \left[L_0 + \sum_{k=1}^{\infty} L_k \cos(k\theta)\right]\omega\frac{di}{d\theta} - i\omega\sum_{k=1}^{\infty} kL_k \sin(k\theta) \tag{10}$$

Equation (8) presents the exact differential equation of the type:

$$P(\theta,i)\,d\theta + Q(\theta,i)\,di = 0 \tag{11}$$

This equation has an analytical solution provided that:

$$\frac{\partial P(\theta,i)}{\partial i} = \frac{\partial Q(\theta,i)}{\partial \theta} \tag{12}$$

The condition (10) is fulfilled by neglecting coil resistance $(R = 0)$.

The equation (8) takes a form:

$$U\left\{\frac{a_0}{2} + \sum_{k=1}^{\infty}\left[a_k \cos(k\theta) + b_k \sin(k\theta)\right]\right\} =$$

$$\left[L_0 + \sum_{k=1}^{\infty} L_k \cos(k\theta)\right]\omega\frac{di}{d\theta} - i\omega\sum_{k=1}^{\infty} kL_k \sin(k\theta) \tag{13}$$

Current analytical solution takes a form:

$$i = \frac{U}{\omega\left(L_0 + \sum_{k=1}^{\infty} L_k \cos k\theta\right)}\left\{(\theta-\alpha)\frac{a_0}{2} + \right.$$

$$\left. + \sum_{k=1}^{\infty}\frac{1}{k}\left[(\sin k\theta - \sin k\alpha)a_k + (\cos k\alpha - \cos k\theta)b_k\right]\right\} \tag{14}$$

Then, instantaneous electromagnetic torque in accordance of (5):

$$m = \frac{1}{2} i^2 \sum_{k=1}^{\infty} kL_k \sin k\theta \tag{15}$$

## 6. RESULTS OF THE CALCULUS

To calculate current and torque waveforms, the parameters of a three phase reluctance motor was used.

On a considered machine following values was measured:

Inductance in aligned position: $L_M = 0{,}8H$
Inductance in unaligned position: $L_m = 0{,}1H$
Number of rotor teeth: $N_r = 40$
Number of the teeth per stator pole: $N_s = 2$
Stator supply voltage: $U = 5V$

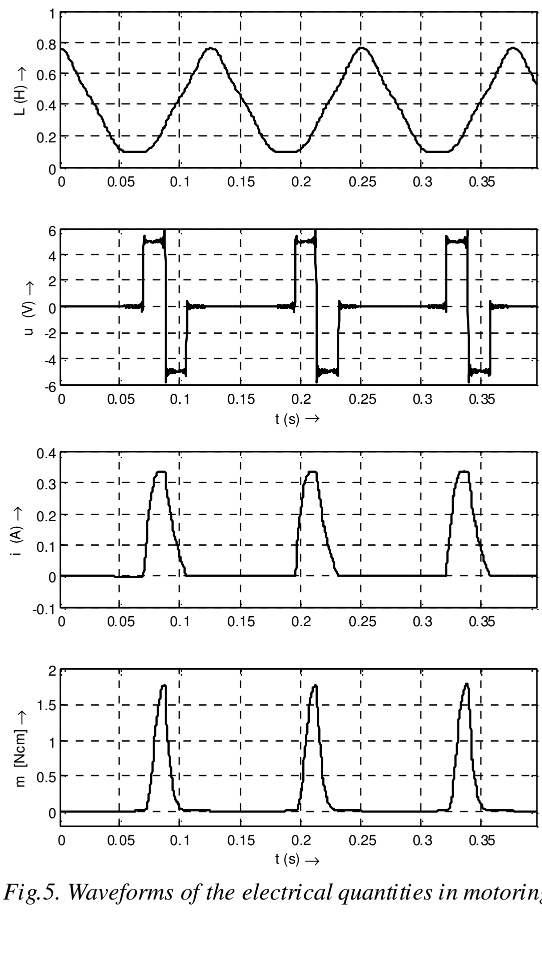


*Fig.5. Waveforms of the electrical quantities in motoring*

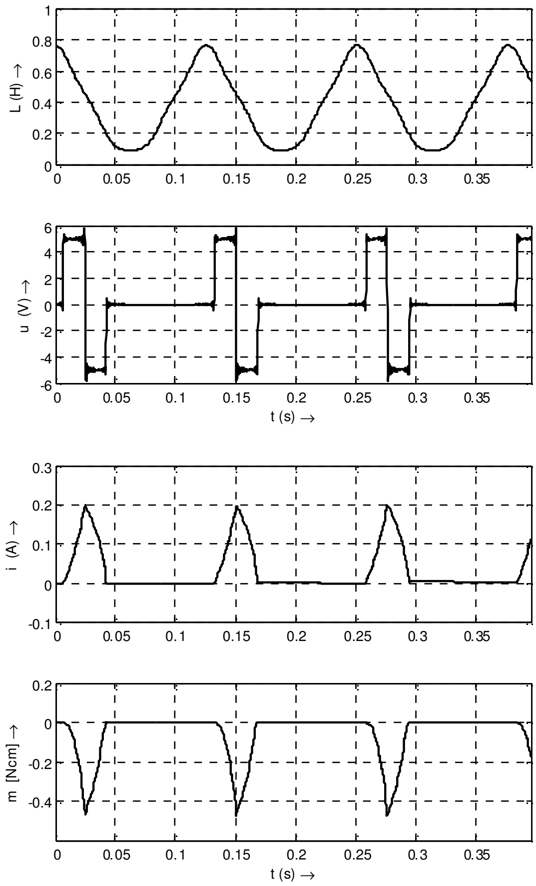


*Fig.6. Waveforms of the electrical quantities in generating*

## 7. CONCLUSION

It is shown an analytical mathematical method for calculus of the phase current and electromagnetic torque of the reluctance stepper motor. Presented method is based on the analytical formularization of the waveform of motor inductance and supply voltage using Fourier series. Calculated motor quantities confirm accuracy of the method.

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The financial support of the Slovak Research and Development Agency under the contract No. APVV-0510-06, is acknowledged.

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