THE SOLITON TRANSMISSIONS IN OPTICAL FIBERS

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Abstract. *The objective of this paper is to familiarize readers with the basic analytical propagation model of short optical pulses in optical fiber. Based on this model simulation of propagation of the special type of pulse, called a soliton, will be carried out. A soliton transmission is especially attractive in the fiber optic telecommunication systems as it does not change a pulses shape during propagating right-down the fiber link to the receiver. The model of very short pulse propagation is based on the numerical solution of the nonlinear Schroedinger equation (NLSE), although in some specific cases it is possible to solve it analytically.*

Keywords

Soliton, ultra short pulses, NLSE, dispersion, Gaussian beam.

1. Introduction

Solitary waves, sometimes simply called soliton, have been a topic of theoretical and experimental study for many years. Though this paper is related to fibre optics, in reality solitary waves exist also in other field of science like hydrodynamics, biology and plasma physics. Historically the one who first observed a soliton wave was James Scott Russel in 1834 when he accidentally noticed in the narrow water canal a smoothly shaped water heap that for his surprise was able to propagate in the canal without a apparent change in its shape a few kilometres along. The essence of propagation of this solitary wave was not a long time understood until appropriate mathematical model was conceived in the 1960's together with a way of solving nonlinear equation with the help of inverse scattering method.

Now let us go back to the field of optics. Generally speaking, there exist two forms of solitary waves, depending on whether the light is being confined in space or time. If the first is the case wave is referred to as spatial soliton or in the second case as temporal soliton. Soliton forming phenomenon stems from nonlinear properties of medium where a particular wave is propagating. Namely, in a field of optics it is Kerr effect that is responsible for optical nonlinearities. In the case of spatial soliton the natural property of light to disperse in space is being proactively compensated by the nonlinearity of the medium in such a way that higher intensity part of an optical beam (typically in the center of Gaussian beam) increase a value of refractive index of medium forming de facto a core of waveguide that is responsible to confine in reverse a dispersed light to the middle of the beam itself. It can be easily intuitively understood that if the self induced nonlinearity is too high the beam will get focused and on the other hand if it is very small or none, beam will disperse in space – a prevailing situation in many cases where a beam does not have enough power density to induce nonlinearity in a medium.

2. Mathematical Modeling of the Solitary Wave

The propagation of light can be precisely described mathematically with Maxwell equations. When equations for magnetic and electric fields are combined together one get [1], [2]:

$$\nabla^2 \vec{E} - \frac{1}{c^2}\frac{\partial^2 \vec{E}}{\partial t^2} = \frac{1}{\varepsilon_0 c^2}\frac{\partial^2 \vec{P}}{\partial t^2}, \tag{1}$$

where c is the speed of light in the vacuum and ε_0 is the vacuum permittivity. The induced polarization P consists of two parts such that [1][2]:

$$\vec{P}(\vec{r},t) = \vec{P}_L(\vec{r},t) + \vec{P}_{NL}(\vec{r},t), \tag{2}$$

where the linear part $\vec{P}_L$ and nonlinear part $\vec{P}_{NL}$ are related to the electric filed by the general relations [1], [2], [3], [4]:

$$\vec{P}_L(\vec{r},t) = \varepsilon_0 \int_{-\infty}^{+\infty} \chi^{(1)}(t-t').\vec{E}(\vec{r},t')dt', \tag{3}$$

$$\vec{P}_{NL}(\vec{r},t) = \varepsilon_0 \iiint_{-\infty}^{+\infty} \chi^{(3)}(t-t_1,t-t_2,t-t_3)\times \vec{E}(\vec{r},t_1)\vec{E}(\vec{r},t_2)\vec{E}(\vec{r},t_3)dt_1dt_2dt_3, \tag{4}$$

where $\chi^{(1)}$ and $\chi^{(3)}$ are the first- and third- order susceptibility tensors.

3. Propagation of Soliton Pulse in Optical Fibers

To better understand a soliton pulse propagation in optical fiber it necessary to set up our modeling on the mathematical expression (1). We will suppose, that a solution for electric filed E have a form [1]:

$$E(r,t) = A(Z,t)F(X,Y)\exp(i\beta_0 Z)\,, \qquad (5)$$

where F(X,Y) is transverse field distribution that corresponds to the fundamental mode of single mode fibre. A(Z,t) is along propagation axis Z and on time t dependent amplitude of the mode. After some math manipulations one can come to the equation that governs pulse propagation in optical fibres [1]:

$$\frac{\partial A}{\partial Z} + \beta_1 \frac{\partial A}{\partial t} + \frac{i\beta_2}{2}\frac{\partial^2 A}{\partial t^2} = i\gamma |A|^2 A\,. \qquad (6)$$

The parameters β_1 a β_2 include the effect of dispersion to first and second orders, respectively. Physically, $\beta_1=1/v_g$, where v_g is group velocity associated with the pulse and β_2 takes into account the dispersion of group velocity. For this reason, β_2 is called the group velocity dispersion (GVD) parameter.

Parameter γ is nonlinear parameter that takes into account the nonlinear properties of a fiber medium. Paraγmeter β_1 is in real case always positive but on the other hand parameters λ_{ZD} and γ can be in some specific case either positive or negative. The parameter β_1 is closely associated in practice with better known parameter called dispersion parameter – D (ps/nm/km). The relation between them is in the form [1]:

$$D = \frac{d}{d\lambda}\left(\frac{1}{v_g}\right) = -\frac{2\pi c}{\lambda^2}\beta_2\,. \qquad (7)$$

As we know, dispersion parameter D is a monotonically increasing function of wavelength, crossing a zero point at wavelength $\lambda_{ZD,}$ which is called a zero chromatic dispersion wavelength. If a system operates with wavelengths above $\lambda_{ZD,}$ where D is positive, β_2 must be negative and a fiber is said to work in anomalous dispersion mode. If a fiber is operated below $\lambda_{ZD,}$ the D is negative and β_2 must be positive. In this case a fiber is said to operate in normal dispersion mode. As regards the nonlinear parameter γ it can be generally either positive or negative, depending on the material of the wave guide. For silica fiber is parameter γ positive but for some other materials it can be negative. More specifically, equation (6) has only two solution, in the form of either dark or bright soliton. The bright soliton corresponds to the light pulse but dark soliton is rather a pulse shaped dip in CW light “background”. In other words, the dark soliton is in a fact negation of the bright soliton. Where there is maximum of light in the bright soliton, there is minimum of the light in the dark soliton and vice versa.

The bright soliton can propagate in only such a waveguide where there is either the positive nonlinearity parameter γ and anomalous dispersion or the negative nonlinear parameter but normal dispersion. For a classical silica fiber the first is the case.

4. The Soliton Pulse and the Simulation of its Propagation in the Optical Fiber

Equation (6) can be normalized in the form:

$$i\frac{\partial u}{\partial z} - \frac{s}{2}\frac{\partial^2 u}{\partial \tau^2} \pm |u|^2 u = 0\,, \qquad (8)$$

using this transforms:

$$\begin{aligned} \tau &= (t-\beta_1 Z)/T_O \\ z &= Z/L_D \\ u &= \sqrt{|\gamma| L_D}\,A \end{aligned}\,, \qquad (9)$$

where T_0 is moving time window width (very often set to the pulse width) and $L_D = T_0^{\,2}/|\beta_2|$ is dispersion length.

Using inverse scattering method reveals that solution of above mentioned equation has a form:

$$u(z,\tau) = N\frac{2}{e^{\tau}+e^{-\tau}}e^{iz/2} = N\mathrm{sech}(\tau)e^{iz/2}\,. \qquad (10)$$

If N is integer, it represents the order of the soliton pulse. Very interesting situation comes when N=1. In this case of first order soliton, the pulse does not change its shape at all as it propagates in optical fiber. In contrast when N is higher than one, pulse shape is not stable and change periodically with soliton period $Z_0 = \frac{\pi}{2}L_D$. At the end of every period Z_0 the soliton resembles its initial simple pulse shape. It is evident that for telecommunication purposes is the soliton of first order most suitable, because in this application is necessary to keep a pulse shape stable.

Parameter N, which defines the soliton order can be further expressed by:

$$N = T_O\sqrt{\frac{P_0\gamma}{|\beta_2|}}\,, \qquad (11)$$

where T_0 [s] corresponds to input pulse width, P_0[W] is pulse peak power β_2 [s^2/m] takes into account group velocity dispersion and γ[$(Wm)^{-1}$] is nonlinear parameter of the fiber material.

I have studied a propagation of soliton pulses within the optical fiber using a simulation tool Optsim form ARTIS In this case a equation (6) is solved numerically using a split-step-Fourier method. The scheme used is shown on Fig. 1. As can be seen, I have used a soliton generator (the most left side icon) that numerically generates sequence of sech(t) pulses. The pulse width was set to 10 ps with period of 400 ps. I have used a standard single mode fiber model with a fiber length of 100 km. I have set up two peak powers of the pulse, namely 100 mW and 166 mW. As pulses satisfy a condition for the soliton shape, it was only necessary to adjust appropriate pulse width and peak power. According to the formula (11) and for the case of first order soliton, we need:

$$N = 1 \Rightarrow P_0 = \frac{|\beta_2|}{T_0^{\,2}\gamma}, \qquad (12)$$

when real values are substituted to the above equation, in particular T_0=10 ps, β_2= -20 ps^2/km and γ=1,2 $W^{-1}km^{-1}$ the adequate power to reach first order soliton regime is approximately 166 mW. In other words if a sech(t) pulse have this peak power one should at least according to theory have a first order soliton that is transmitted through a fiber unchanged in shape. To verify this I have used in above model a fiber without a loss, it is possible of course only in simulations not in the practice. In the reality it would be necessary to deploy optical amplifiers (like EDFA or Raman, or both in combination) to overcome the loss in the real fiber link and keep the power of the soliton in required limits, otherwise the pulse will start to spread again, the effect of nonlinearity will be not so strong to effectively suppress unwelcome influence of the fiber chromatic dispersion.

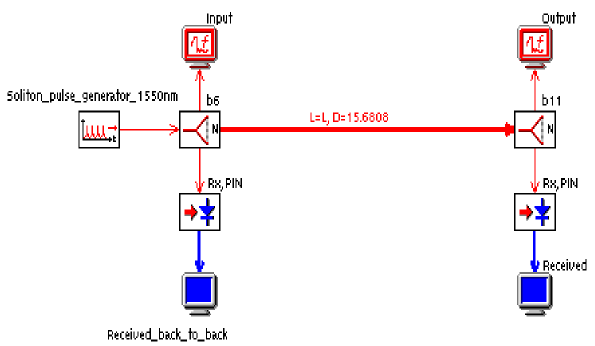

Fig. 1: Simulation of soliton in fiber.

To illustrate creation of the soliton I have used the scheme on Fig. 1. The pulse shape can be seen on Fig. 2. The right couple of overlapping (higher one above other) pulses corresponds to the input. These pulses were in sequence sent to 100 km section of the single mode fiber link.

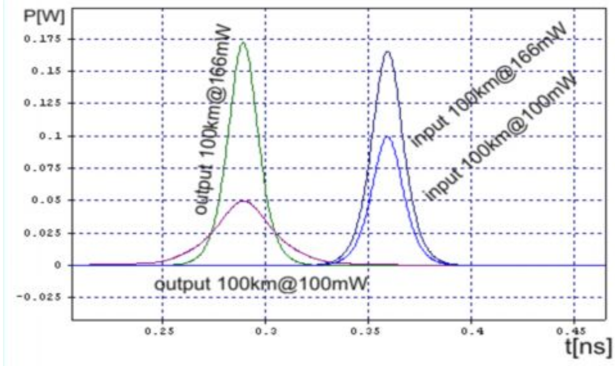

Fig. 2: Simulation of soliton in fiber.

The one with the lower power (100 mW) relates to the lower output pulse that is shown at the left side of the picture. It is evident that in this case the pulse undergoes spreading caused by chromatic dispersion and peak power is not yet high enough to compensate dispersion. On the other hand, if one increase optical power it is possible to get to the point, where the nonlinearity entirely compensate dispersion and the pulse propagates in the fiber link without any change in the shape, except a small increase in its amplitude. It is worth to mention right now, that this increase in amplitude is temporal phenomenon because if one would have study the pulse propagation more carefully it would be revealed that its peak oscillates until reaching a point of steady amplitude. This happens mostly in cases when the input pulse does not resemble the exact soliton form. The beauty of a soliton is among other things in its ability to reassemble original shape despite of some disturbing factors acting upon it.

5. Conclusion

In this paper a very basic analysis of solition transmission and its dynamics in the optical fiber was performed. It has been shown by simulation that bright soliton can form in classical optical single mode fiber and in the case of zero loss it can propagate without a change of its shape. As result it is possible to overcome issue with influence of the dispersion on pulse spreading and achieve a much longer transmission distances and also increase line capacity. However, the application of soliton transmission is not entirely without problems. Soliton pulses should be apart in time considerably to avoid excessive overlapping. If soliton's overlap is not sufficiently suppressed solitons will have an effect on the transmission of themselves in a way to cause individual soliton's group velocity to vary along the line and resulting in increased system jitter.

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References

[1] KIVSHAR, Y. S.; AGRAWAL, G. P. *Optical Solitons: From Fibers to Photonic Crystals*. *San Diego*. Academic Press, 2003. 540 s. ISBN 0-12-410590-4.

[2] PORSEZIAN, K.; KURIAKOSE, V.C. *Optical Solitons: Therotical and Experimentak challenges*. 1st edition. Berlin: Springer, 2003. 406 s. ISBN 3-540-00155-7.

[3] AGRAWAL, G. P. *Applications on Nonlinera Fiber Optics*. San Diego: Academic Press, 2001. 458 s. ISBN 0-12-045144-1.

[4] GAGLIARDI, R. M.; KARP, S. *Optical Communications*. 2nd edition. New York : Wiley, 1995. 347 s. ISBN 0-471-54287-3.

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Leos BOHAC received the M.S. and Ph.D. degrees in electrical engineering from the Czech technical university in Prague in 1992 and 2001, respectively. From 1992 he has been teaching Optical communication systems and Data networks at the same university. His research interest is an application of the high speed optical transmission systems in a data network. He has also participated in optical research projects in CESNET, the academic data network provider, to help implement long haul high speed optical research network.