Investigation on Superior Performance by Fractional Controller for Cart-Servo Laboratory Set-Up

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Abstract. *In this paper, an investigation is made on the superiority of fractional PID controller ($PI^{\alpha}D^{\beta}$) over conventional PID for the cart-servo laboratory set-up. The designed controllers are optimum in the sense of Integral Absolute Error (IAE) and Integral Square Error (ISE). The paper contributes in three aspects: 1) Acquiring nonlinear mathematical model for the cart-servo laboratory set-up, 2) Designing fractional and integer order PID for minimizing IAE, ISE, 3) Analyzing the performance of designed controllers for simulated plant model as well as real plant. The results show a significantly superior performance by $PI^{\alpha}D^{\beta}$ as compared to the conventional PID controller.*

Keywords

Cart-servo, fractional PID, IAE, ISE.

1. Introduction

Fractional calculus [1] has recently found new applications in control engineering resulting in an area popularly known as 'Fractional Order Control (FOC)'. FOC is nothing but designing the controllers which are governed by fractional order the differential equations. The compact form expressions of these controllers possess easily tunable characteristics for meeting stringent loop performance [2], [3], [4], [5], [6].

The tuning of three-parameter fractional order controllers such as PI^{α}, $[\mathrm{PI}]^{\alpha}$, PD^{β}, $[\mathrm{PD}]^{\beta}$ has been addressed in the literature [7], [8], [9], [10], [11]. The formulation in these works consists of a set of three equations which are solved analytically or graphically.

Literature also covers tuning of five-parameter $\mathrm{PI}^{\alpha}\mathrm{D}^{\beta}$ controller to minimize certain performance indices such as Integral Absolute Error (IAE), Integral Square Error (ISE), etc. [12], [13]. This is an unconstrained, five-dimensional and multi-modal optimization problem in which the objective function is optimized with respect to five parameters. The works in [12], [13] have considered linear plants. However, one can also design IAE, ISE minimizing fractional controllers for the given nonlinear plant model. A few early works in this regard are seen in the literature [14], [15], [16], [17] in which the superiority of fractional order controllers over integer ones is investigated.

In the present paper, we explore further in this direction to examine the fractional superiority for cart-servo lab set-up which contains a few nonlinear elements.

The major contributions of this paper are as follows:

- The mathematical model of the cart-servo lab set-up is obtained which is further validated by performing a model-matching test.
- For the acquired model, optimum fractional and integer order PID controllers are designed so as to minimize performance indices such as IAE and ISE.
- The designed controllers are analyzed in detail for their performance with the plant model as well as real plant to examine the superiority of fractional controllers.

Organization of the paper: Section 2 presents mathematical modelling of the cart-servo lab setup and its validation using model-matching test. In section 3, preliminaries of the fractional order control are discussed and also the controller design problem for minimizing performance indices such as IAE, ISE is explained. Section 4 demonstrates the design of integer and fractional order PID controllers to meet the control requirements. Also, the performance of designed controllers is discussed and compared in section 4. Finally, section 5 provides the concluding remarks.

2. Mathematical Modelling of Cart-Servo Lab Set-Up

2.1. Plant Description

The original cart-pendulum lab set-up designed by Feedback Instruments, UK consists of a cart moving along a 1 metre long track [18]. The cart has a shaft to which the pendulum is attached. The cart can move back and forth causing the pendulum to swing.

For the cart-servo control purpose intended for the current paper, the pendulum is detached from the above set-up as shown in Fig. 1. The movement of the cart is caused by pulling the belt in two directions by the DC motor attached at one end of the rail. The control task is to attain the desired cart position on the rail which is realized by controlling the input voltage to the motor.

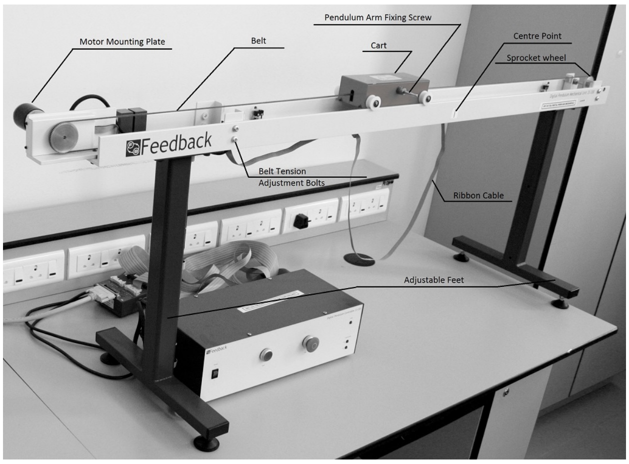

Fig. 1: Cart-servo plant experimental set-up.

2.2. Model Identification

In model-based controller tuning approach, it is essential to have the sufficiently captured mathematical model for the real plant dynamics. It is carried out as explained below.

- The equations governing cart-servo plant dynamics are [18]:

$$V_a - K_b\omega_m = R_m I_a + L_m \frac{dI_a}{dt}, \tag{1}$$

$$T_m = K_t I_a, \tag{2}$$

$$T_{load} = T_m - J_m\alpha_m - B_m\omega_m, \tag{3}$$

$$F_{load} = T_{load} g_r, \tag{4}$$

$$F_{load} = M_c a_c + B_c v_c + F_{frict}. \tag{5}$$

- Incorporating Eq. (1), Eq. (2), Eq. (3), Eq. (4), Eq. (5) along with the current-loop type of power amplifier dynamics [18], we construct the complete mathematical model of the cart-servo set-up as shown in Fig. 2. In Fig. 2, the gray shaded blocks are the nonlinear elements present in the system. Also, the cart auxiliary velocity v_{cd} has been obtained from cart velocity v_c to eliminate the 'algebraic loop' while simulating the mathematical model. (Note: Table of parameters and the table of variables have been given in Tab. 1 and Tab. 2.)

- To define the nonlinear relation between [F_{frict}, Reset] and [F_{load}, v_{cd}], we propose the following embedded MATLAB code (Refer the block, 'Cart Friction Model' in Fig. 2):

$function[F_{frict}, Reset] = cartfrict(F_{load}, v_{cd})$
$fr = 1.68; Mc = 2.3; Ts = 0.0001;$
$Fc = fr * sign(v_{cd}); \%fr = frictioncoefficient,$
$Ts = SamplingInterval$
if $v_{cd} == 0$
$Fc = fr * sign(F_{load});$
end
if $v_{cd} == 0$
if $abs(F_{load}) < fr$
$Reset = 1; F_{frict} = F_{load};$
else
$Reset = 0; F_{frict} = fr * sign(F_{load});$
end
else
$vc2 = v_{cd} + (F_{load} - F_c)/Mc * Ts;$
if$((sign(vc2 * v_{cd}) < 0)\&\&(sign(F_{load} * vc2) < 0))$
$||((sign(vc2 * v_{cd}) < 0)\&\&$
$((sign(F_{load} * vc2) > 0)\&\&(abs(F_{load}) < fr)))$
$Reset = 1; F_{frict} = F_{load};$
else
$Reset = 1; F_{frict} = F_{load};$
end
end

2.3. Model-Matching

In order to validate the acquired model as shown in Fig. 2, we perform a model-match test. For this purpose, a sweep signal of amplitude 0.2 is generated [1]. The sweep signal is given as an input to a closed loop

[1] The sweep signal is a composite signal which is constructed by the sine-waves of different frequencies such that the time instance at which one sine-wave ends, the other one begins.

Tab. 1: Table of parameters.

Description	Symbol	Unit	Value
Power amplifier input voltage saturation limit	V_c	V	2.5
Power amplifier supply voltage saturation limit	V_s	V	24
Power amplifier current loop gain	K_1	-	$\frac{1}{35}$
Power amplifier forward gain	K_2	-	200
Motor armature resistance	R_m	Ω	2.5
Motor armature inductance	L_m	H	0.0025
Motor back-emf constant	K_b	V/(rad/sec)	0.05
Motor torque constant	K_t	N-m/A	0.05
MI of motor rotating assembly	J_m	Kg-m^2	0.000014
Viscous damping coefficient of motor shaft	B_m	N-m/(rad/sec)	0.000001
Rotary to linear motion conversion ratio	g_r	m^{-1}	600
Viscous damping coefficient of cart	B_c	N-m/(rad/sec)	0.00005
Mass of cart	M_c	Kg	2.3
Second order filter natural frequency	ω_f	rad/sec	2215.7
Second order filter damping factor	ζ_f	-	0.7

Tab. 2: Table of variables.

Description	Symbol
Armature voltage	V_a
Armature current	I_a
Motor torque	T_m
Load torque	T_{load}
Load force	F_{load}
Friction force	F_{frict}
Motor shaft angular velocity	ω_m
Motor shaft angular acceleration	α_m
Cart position	x_c
Cart velocity	v_c
Cart auxillary velocity	v_{cd}
Cart acceleration	a_c

system which contains simulated/real plant with unity gain controller.

The response is as shown in Fig. 3. It is seen from Fig. 3 that the responses for simulated as well as real plant are close enough to confirm sufficient capture of plant dynamics in its mathematical model.

The response to the sweep signal is composed of responses to each (frequency) sine-wave in the corresponding time-intervals. By considering the fundamental harmonic in such an output response corresponding to each sine-wave, one can construct the frequency response of the closed loop system. We obtain the closed loop frequency responses with real and simulated plant as presented in Fig. 4.

The mathematical model (refer Fig. 2) consisting of elements such as Cart-Friction Model, current loop power amplifier, etc. sufficiently captures the lower mode dynamics of cart-servo which is in the required control-passband frequency range. High frequency phenomena such as mechanical vibrations, switching in power amplifier circuits, high frequency noise signal etc. have not been taken into consideration while modelling. Therefore, one can see in Fig. 4 that the frequency responses with simulated and real plant match closely for the lower range of frequency while there is a mismatch between these responses at the higher frequency side.

3. Basics of Fractional Controllers and Controller Design Problem

3.1. Preliminaries of Fractional Controllers

1) Fractional Calculus

Conventional calculus deals with integer order differentiation and integration. Generalization of conventional calculus so as to consider differentiation and integration of any order (not necessarily integer) leads to 'Fractional Calculus (FC)' [1]. In FC, the fundamental differ-integration operator ${}_aD_t^\alpha$ (where a and t are the limits of the operation) is defined as [2]:

$$ {}_a\mathrm{D}_t^\alpha = \begin{cases} \frac{d^\alpha}{dt^\alpha} & \alpha > 0 \\ 1 & \alpha = 0 \\ \int_a^t (d\tau)^{-\alpha} & \alpha < 0 \end{cases}, \qquad (6)$$

where α is the order of the operation, generally $\alpha \in \mathbb{R}$ but α could also be a complex number.

Out of many definitions of fractional differ-integration in FC, the popular ones are [2]:

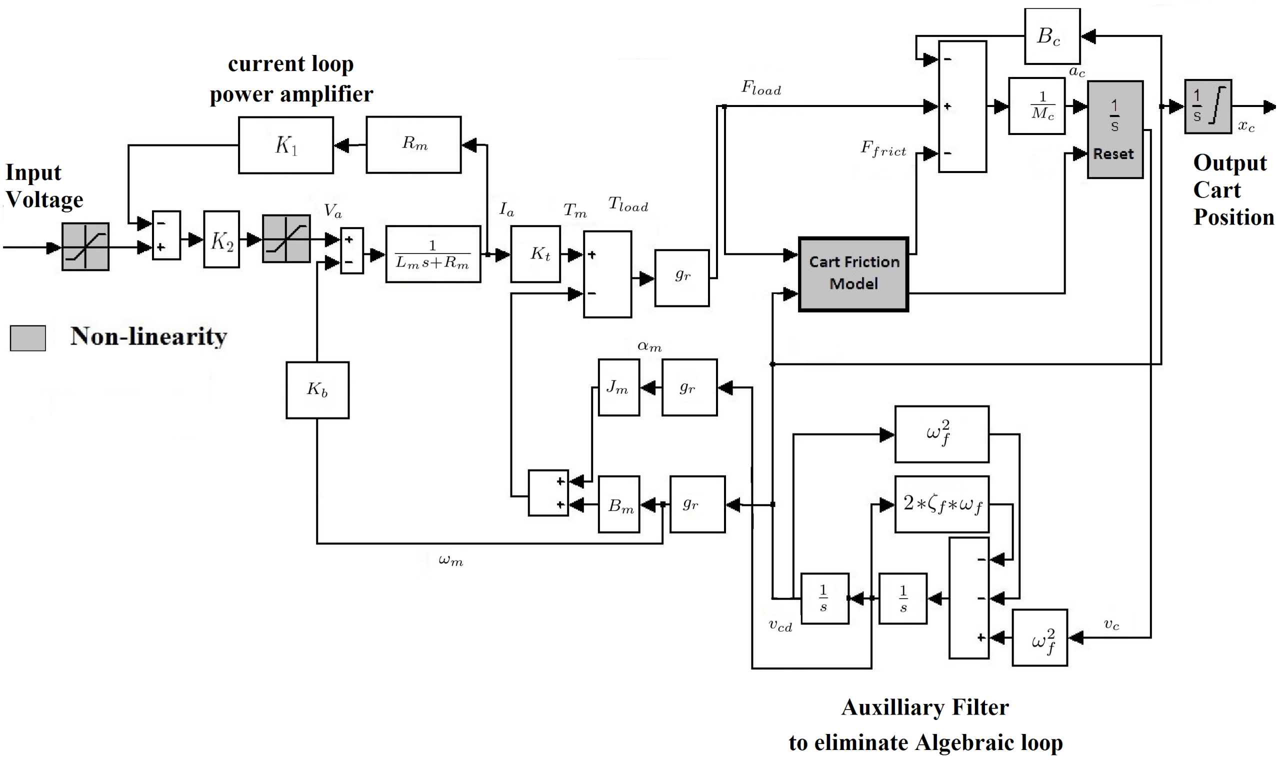

Fig. 2: Mathematical model of cart-servo plant.

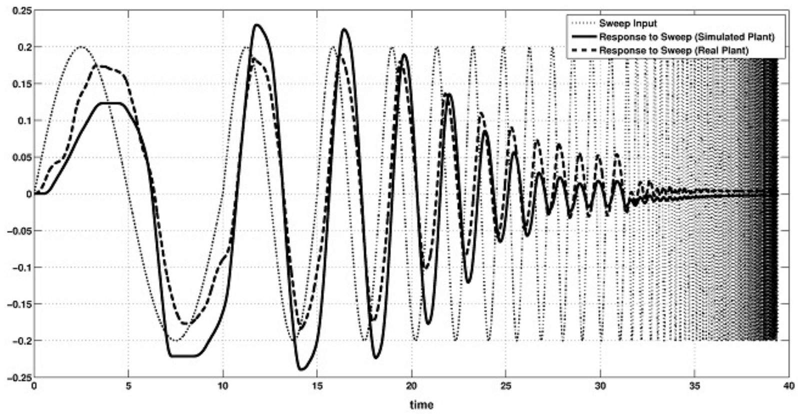

Fig. 3: Closed loop response with real and simulated plant to sweep signal.

- Grunwald-Letnikov Definition:

$$_{\alpha}\mathrm{D}_t^{\alpha} f(t) = \lim_{h\to 0} \frac{1}{h^{\alpha}} \sum_{j=0}^{\left[\frac{t-a}{h}\right]} (-1)^j \binom{\alpha}{j} f(t-jh), \quad (7)$$

where, $\left[\frac{t-a}{h}\right]$ truncates $\frac{t-a}{h}$ to an integer.

- Riemann-Liouville (R-L) Definition:

$$_{\alpha}\mathrm{D}_t^{\alpha} f(t) = \frac{1}{\Gamma(n-\alpha)} \left(\frac{d}{dt}\right)^n \int_a^t \frac{f(\tau)}{(t-\tau)^{\alpha-n+1}} d\tau, \quad (8)$$

where n is an integer, a is a real number, and α satisfies $(n-1) \leq \alpha < n$.

- Caputo Definition:

$$_{a}D_t^{\alpha} f(t) = \frac{1}{\Gamma(n-\alpha)} \int_a^t \frac{f^{(n)}(\tau)}{(t-\tau)^{\alpha-n+1}} d\tau. \quad (9)$$

2) Fractional Order Transfer Function Model

The equation of Laplace transform for the defined fractional-order operator is [2]:

$$L(_{a}D_t^{\alpha} f(t)) = s^{\alpha} F(s), \quad (10)$$

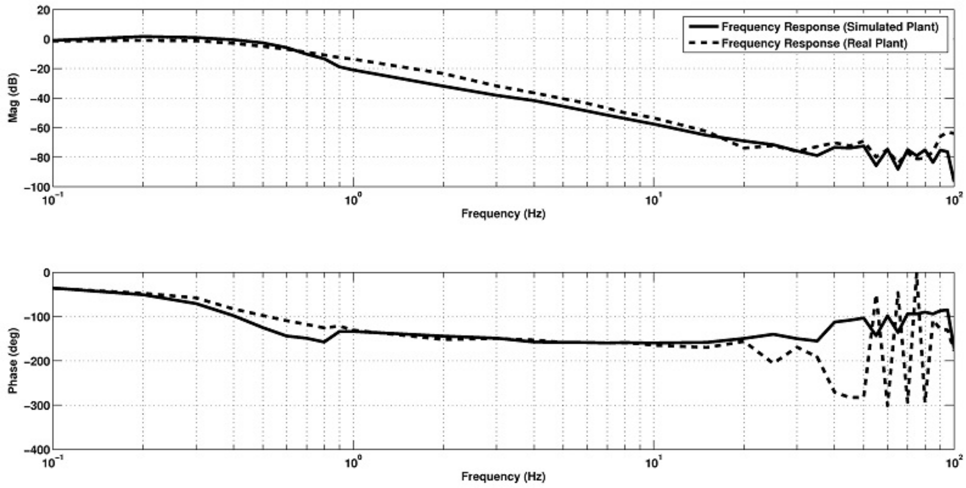

Fig. 4: Closed loop frequency response with real and simulated plant.

with zero initial conditions.

Linear time invariant fractional model of a system with input u, and output y takes the following form [2]:

$$a_n D^{\alpha_n} y(t) + a_{n-1} D^{\alpha_{n-1}} y(t) + ... + a_0 D^{\alpha_0} y(t) = b_m D^{\beta_m} u(t) + b_{m-1} D^{\beta_{m-1}} u(t) + ... + b_0 D^{\beta_0} u(t), \quad (11)$$

where, a_i, α_i $(i = 0, 1, \ldots, n)$, b_k, β_k $(k = 0, 1, \ldots, m)$ are real constants. n and m are positive integers.

Therefore, Laplace transform on both sides (assuming zero initial conditions) results into the following transfer function:

$$\frac{Y(s)}{U(s)} = \frac{b_m s^{\beta_m} + b_{m-1} s^{\beta_{m-1}} + ... + b_0 s^{\beta_0}}{a_n s^{\alpha_n} + a_{n-1} s^{\alpha_{n-1}} + ... + a_0 s^{\alpha_0}}. \quad (12)$$

3) Fractional Order Controller

From control engineering point of view, the application of FC can be in either system modelling or controller design. The typical fractional order controllers $C(s)$ found in the literature are as follows:

Fractional order proportional-integral controller, which is of two types [9]:

- PI^{α}

$$C(s) = K_p \left(1 + \frac{K_i}{s^{\alpha}}\right). \quad (13)$$

- $[\mathrm{PI}]^{\alpha}$

$$C(s) = K_p \left(1 + \frac{K_i}{s}\right)^{\alpha}, \quad (14)$$

with $\alpha = 1$, we get Integer PI of the form: $C(s) = K_p \left(1 + \frac{K_i}{s}\right)$.

Fractional order proportional-derivative controller, which is of two types [7], [8]:

- PD^{β}

$$C(s) = K_p \left(1 + K_d s^{\beta}\right). \quad (15)$$

- $[\mathrm{PD}]^{\beta}$

$$C(s) = K_p \left(1 + K_d s\right)^{\beta}, \quad (16)$$

with $\beta = 1$, we get Integer PD of the form: $C(s) = K_p (1 + K_d s)$.

Fractional order proportional-integral-derivative controller [2]:

- $\mathrm{PI}^{\alpha}\mathrm{D}^{\beta}$

$$C(s) = K_p \left(1 + \frac{K_i}{s^{\alpha}} + K_d s^{\beta}\right), \quad (17)$$

with $\alpha = 1, \beta = 1$, we get Integer PID of the form: $C(s) = K_p \left(1 + \frac{K_i}{s} + K_d s\right)$.

3.2. Design of Optimum Controller to Minimize IAE, ISE

The typical unity feedback control system is shown in Fig. 5. $r(t)$, $e(t)$, $u(t)$, and $y(t)$ denote reference input, error, controller output, and plant output respectively.

The following performance indices are considered:

- Integral Absolute Error (IAE):

$$J = \int_0^{\infty} |e(t)| dt. \quad (18)$$

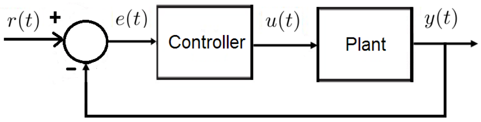

Fig. 5: Typical unity feedback control system.

- Integral Square Error (ISE):

$$J = \int_0^\infty |e(t)|^2 dt. \quad (19)$$

Let $e(kT)$ be the sampled value of the error $e(t)$ at an instant (kT), where T is the sampling interval. k=0,1,...,N. For the given T, N is an integer which depends on the time span considered for computing $e(t)$. The following cost functions are considered corresponding to the performance indices defined in Eq. (18), Eq. (19):

- IAE cost function:

$$J_c = \sum_{k=0}^{N} |e(kT)|T. \quad (20)$$

- ISE cost function:

$$J_c = \sum_{k=0}^{N} |e(kT)|^2 T. \quad (21)$$

Each performance index emphasizes different aspects of the system response [19]. Large errors contribute more in ISE than IAE. Consequently, the controller tuned for minimizing ISE ensures lower overshoot in the transient response than IAE minimizing controller. The ISE, however tends to give larger settling time.

For the optimum performance of control system, controller parameters are tuned by minimizing the selected performance index.

4. Design and Performance Analysis of Integer and Fractional PID for Cart-Servo

Mathematical model of the cart-servo plant (as developed in Section 2) is considered for designing PI$^\alpha$D$^\beta$ and PID controllers (refer Eq. (17) for the controller structure). The controller design problem for minimizing IAE and ISE (as discussed in Section 3) is solved numerically with MATLAB using fminsearch() function.

While tuning, a step input of 0.2 m is given to the closed loop system for 10 sec. The sampling interval is taken as 0.001 sec. The search space for the controller is limited by assigning certain bounds to its parameters. Fractional order integration and differentiation are ensured by choosing $\alpha \in (0,1)$ and $\beta \in (0,1)$ respectively. In PI$^\alpha$D$^\beta$ controller, if $\alpha = 1$ and $\beta = 1$, it becomes the PID controller. Therefore, to eliminate the case of PID while tuning PI$^\alpha$D$^\beta$, the values $\alpha = 1$ and $\beta = 1$ are not included in the bounds for α and β. The bounds for K_p, K_i and K_d ($K_p \in (0,15]$, $K_i \in (0,5]$, $K_d \in (0,1]$) have been suitably chosen by referring their typical values for the design examples given in the manual [18]. However, one is free to choose any other valid bounds. For the selected bounds, the emphasis is on the investigation of possible superior performance by PI$^\alpha$D$^\beta$ over PID.

Oustaloup [20] approximated transfer function model of the PI$^\alpha$D$^\beta$ controller is considered for the simulation. The order of Oustaloup approximation is taken as 7 and the approximation is valid over the frequency range [0.001,1000] rad·s^{-1}.

After the design, controllers are tested with simulated as well as real plant. The results are presented in Tab. 3 and Tab. 4. The following conclusions are derived based on Tab. 3 and Tab. 4:

- For simulated plant, fractional PID reduces J_{IAE} by nearly 65 % and J_{ISE} by nearly 62 % as compared to integer PID.

- For the real plant case, fractional PID reduces J_{IAE} by nearly 75 % and J_{ISE} by nearly 65 % as compared to the integer PID.

- The differences in the performance index values obtained for simulated and real plants are due to slight imperfections in the captured plant dynamics.

Figure 6 presents cart-position and error signals for the closed loop system with simulated plant and fractional/integer PID controller tuned for IAE minimization. A step signal of amplitude 0.2 m is given for 10 sec duration as input. The corresponding response with real plant is shown in Fig. 7. The responses for ISE minimization case are shown in Fig. 8 and Fig. 9.

From Fig. 6, Fig. 7, Fig. 8, Fig. 9, we observe that the cart-position response with fractional PID shows significantly smaller rise time, settling time, peak overshoot as compared to the integer PID. This means that the performance with fractional PID is far superior over its integer counter part for cart-servo lab set-up.

Tab. 3: IAE for designed controllers with simulated and real plants.

Performance Index		Controller		Reduction in Performance Index with $PI^{\alpha}D^{\beta}$ over PID (%)
		PID	$PI^{\alpha}D^{\beta}$	
		$K_p = 14.9815$, $K_i = 2.7509$, $K_d = 0.5$	$K_p = 1.3124$, $K_i = 4.9906$, $\alpha = 0.1874$, $K_d = 2.9322$, $\beta = 0.5452$	
IAE	$J_{simulated}$	0.1562	0.0551	64.725
	J_{real}	0.1636	0.0393	75.978

Tab. 4: ISE for designed controllers with simulated and real plants.

Performance Index		Controller		Reduction in Performance Index with $PI^{\alpha}D^{\beta}$ over PID (%)
		PID	$PI^{\alpha}D^{\beta}$	
		$K_p = 14.9762$, $K_i = 3.2929$, $K_d = 0.5001$	$K_p = 4.999$, $K_i = 4.7835$, $\alpha = 0.0535$, $K_d = 0.8685$, $\beta = 0.7384$	
ISE	$J_{simulated}$	0.0120	0.0046	61.667
	J_{real}	0.0128	0.0045	64.844

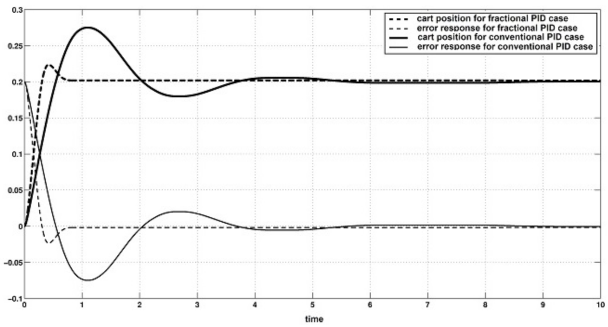

Fig. 6: Step and error response with simulated plant (IAE minimization).

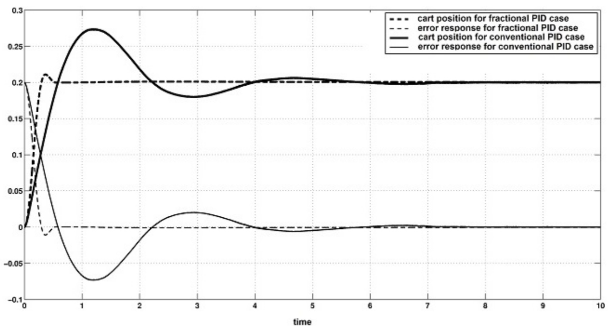

Fig. 7: Step and error response with real plant (IAE minimization).

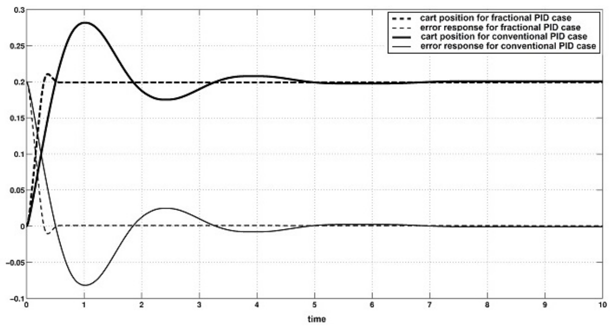

Fig. 8: Step and error response with simulated plant (ISE minimization).

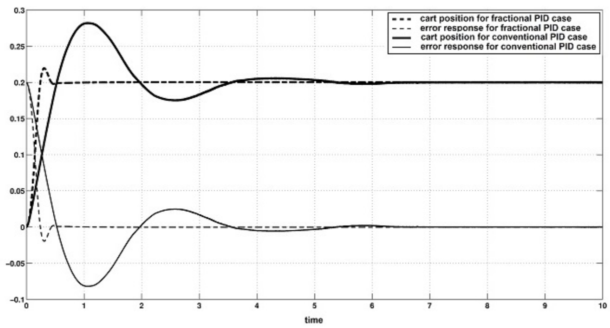

Fig. 9: Step and error response with real plant (ISE minimization).

5. Conclusion

The paper presented the ability of fractional PID controller ($PI^{\alpha}D^{\beta}$) to produce superior performance over conventional PID for cart-servo lab set-up. For this purpose, these controllers were designed for the acquired mathematical model of the plant so as to minimize performance indices, IAE and ISE. The designed controllers were tested with the simulated as well as real plant. It was observed that the fractional PID outperformed integer order PID by significantly reducing the performance indices (more than 60 % in each case).

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